General Conditions for Scale-invariant Perturbations in Expanding Universe

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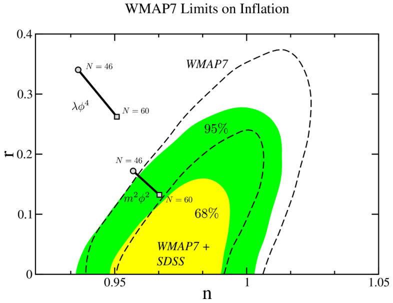
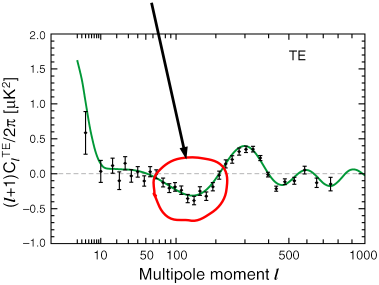
G.Geshnizjani, W.Kinney, A.M, JCAP 1111 (2011) 049

What do we observe in CMB?

Scale-invariant perturbations
$n_s = 0.963 \pm 0.014$

Super-Hubble correlated fluctuations at recombination





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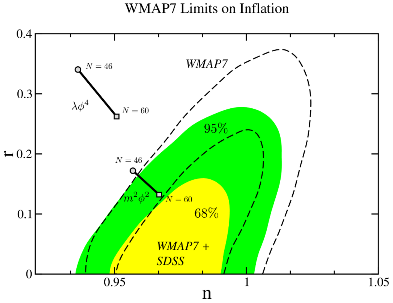
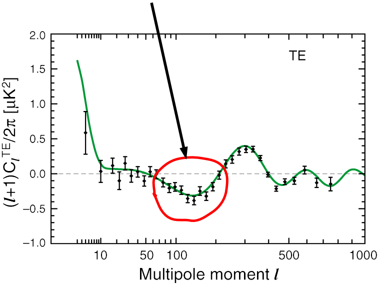




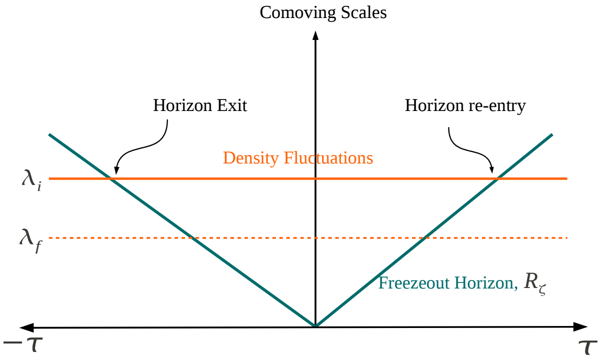
What general conclusions can be made about the physics of early universe?

Result in short

In an *expanding* universe, to obtain perturbations consistent with observation at least one of these three conditions must be satisfied:

- Accelerated expansion, i.e. inflation.
- Super-Planckian energy density.
- Super-luminal speed of sound.

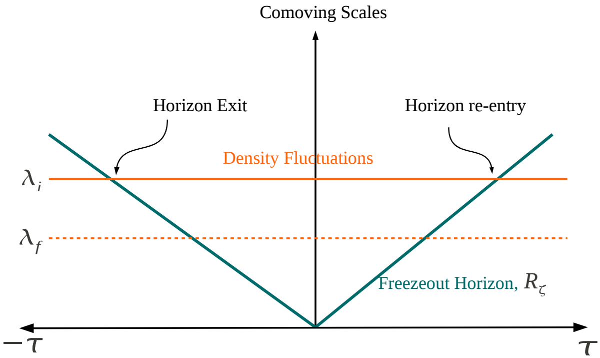
Generation of perturbations, contd.



A key point: **shrinking comoving horizon**

Can we have shrinking comoving horizon without inflation?

Generation of perturbations, contd.



A key point: **shrinking comoving horizon**
Can we have shrinking comoving horizon without inflation?

Canonical Case

In terms of Mukhanov-Sasaki variables:

$$v \equiv z\zeta, \qquad z = a\sqrt{2\epsilon},$$

the mode equation in Fourier space is given by:

$$v_k'' + (k^2 - \frac{z''}{z})v_k = 0.$$

Scale invariance:

$$\frac{z''}{z} = \frac{2}{\tau^2} \equiv R_\zeta^{-2}$$

Two horizons

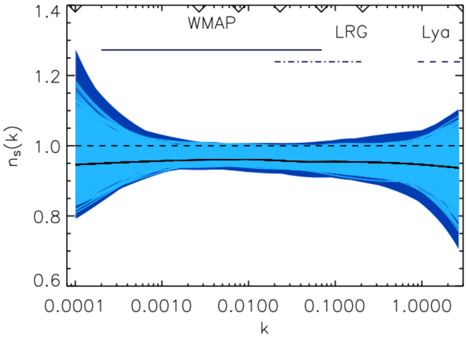
- Hubble horizon: $$R_H = \frac{1}{aH} = \frac{a}{a'}$$
- Freezeout horizon: $$R_\zeta = \sqrt{\frac{a\sqrt{\epsilon}}{\left(a\sqrt{\epsilon}\right)''}}$$

$$\text{Slow-roll inflation} \quad \rightarrow \quad R_\zeta, R_H \propto \tau$$

$$\text{In general} \quad R_H \neq R_\zeta$$

Khoury and Miller, PhysRevD.84.023511

What we know



Peiris and Verde, PhysRevD.81.021302

Power spectrum is scale-invariant over at least a factor of 1000 in wavelength

Assumptions

- Non-accelerated expansion, $\epsilon > 1$
- Observational constraints:
  - Scale-invariant spectrum of perturbations over at least three decades in k-space: $\lambda_i \gtrsim 1000\ \lambda_f$
  - Super-Hubble correlated perturbations: $\lambda_f\left(\tau_f\right) > R_H\left(\tau_f\right)$

Super-Planckian energy density

- Horizon crossing:

$$\lambda_i(\tau_i) = R_\zeta(\tau_i) = |\tau_i|$$
$$\lambda_f(\tau_f) = R_\zeta(\tau_f) = |\tau_f| > R_H(\tau_f)$$

- CMB+LSS: $\lambda_i > 1000\lambda_f$

$$\boxed{\frac{\tau_f - \tau_i}{R_H(\tau_f)} > 1000}$$

Super-Planckian energy density, contd.

Combined with continuity equation:

$$\dot{\rho} = -2\epsilon H \rho$$

non-accelerating expansion with shrinking freezout horizon:

$$\ln \frac{\rho_i}{\rho_f} > 2\frac{\tau_f - \tau_i}{R_H(\tau_f)} > 2000$$

Lower bound on ρ_f given by the lower bound on reheat temperature: $\rho_f > \rho_r > (100 \text{ MeV})^4$

$$\boxed{\rho_i \gg M_{\rm Pl}^4}$$

Non-canonical case

Quadratic action for curvature perturbations:

$$S_2 = \frac{M_{pl}^2}{2}\int dx^3 d\tau\ z^2\left[\left(\frac{d\zeta}{d\tau}\right)^2 - c_s(\tau)^2(\nabla\zeta)^2\right]$$

Through a time transformation $dy = c_s d\tau$:

$$S_2 = \frac{M_{pl}^2}{2}\int dx^3 dy\ q^2\left[\left(\frac{d\zeta}{dy}\right)^2 - (\nabla\zeta)^2\right]$$

where:

$$z \equiv \frac{a\sqrt{2\epsilon}}{c_s} \qquad q \equiv \frac{a\sqrt{2\epsilon}}{\sqrt{c_s}}$$

Khoury and Piazza, JCAP 0907:026,2009

Non-canonical case, contd.

In terms of Mukhanov-Sasaki variables:

$$v \equiv q\zeta \qquad\qquad z = \frac{a\sqrt{2\epsilon}}{\sqrt{c_s}}$$

the mode equation in Fourier space is given by:

$$v_k'' + (k^2 - \frac{q''}{q})v_k = 0$$

Scale-invariance:

$$\frac{q''}{q} \sim \frac{2}{y^2}$$

Super-luminal speed of sound

- Horizon crossing:

$$\begin{aligned} \lambda_i(\tau_i) &= R_\zeta(\tau_i) = |y_i| \\ \lambda_f(\tau_f) &= R_\zeta(\tau_f) = |y_f| > R_H(y_f) \end{aligned}$$

$$y_f - y_i = \int_{\tau_i}^{\tau_f} c_s d\tau = \bar{c}_s(\tau_f - \tau_i)$$

- CMB+LSS: $\lambda_i > 1000\lambda_f$

$$\boxed{\frac{\bar{c}_s(\tau)(\tau_f - \tau_i)}{R_H(\tau_f)} > 1000}$$

Super-luminal speed of sound, contd.

From continuity equation:

$$\ln \frac{\rho_i}{\rho_f} > 2\frac{y_f - y_i}{R_H(\tau_f)} > \frac{2000}{\bar{c}_s}$$

For:

$$\begin{aligned} \rho_i &\leq M_P^4 \\ \rho_f &\geq (100Mev)^4 \end{aligned}$$

$$\boxed{\bar{c}_s > 10}$$

Conclusion

In order to generate the perturbations consistent with observations at least one of the three conditions must be satisfied:

- Accelerated expansion, i.e. inflation.
- Super-Planckian energy density.
- Super-luminal speed of sound.