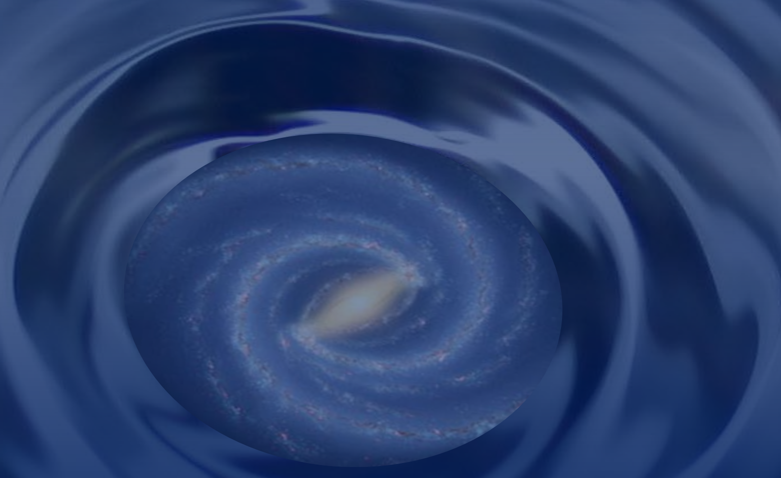
Partially Massless Gravity



Carnegie Mellon University
Workshop on Cosmic Acceleration

Claudia de Rham
Aug. 24th 2012

1206.3482 with S. Renaux-Petel

Why Modify Gravity ?

- There is little doubt that GR breaks down at high energy (Mpl ? TeV ???)

- Does gravity break down in the IR ?

Late time acceleration & CC problem	⟷	First signs of the breakdown of GR on cosmological scales

Tackling Dark Energy

- Either ignore the vacuum energy from particle physics and find another source for the acceleration

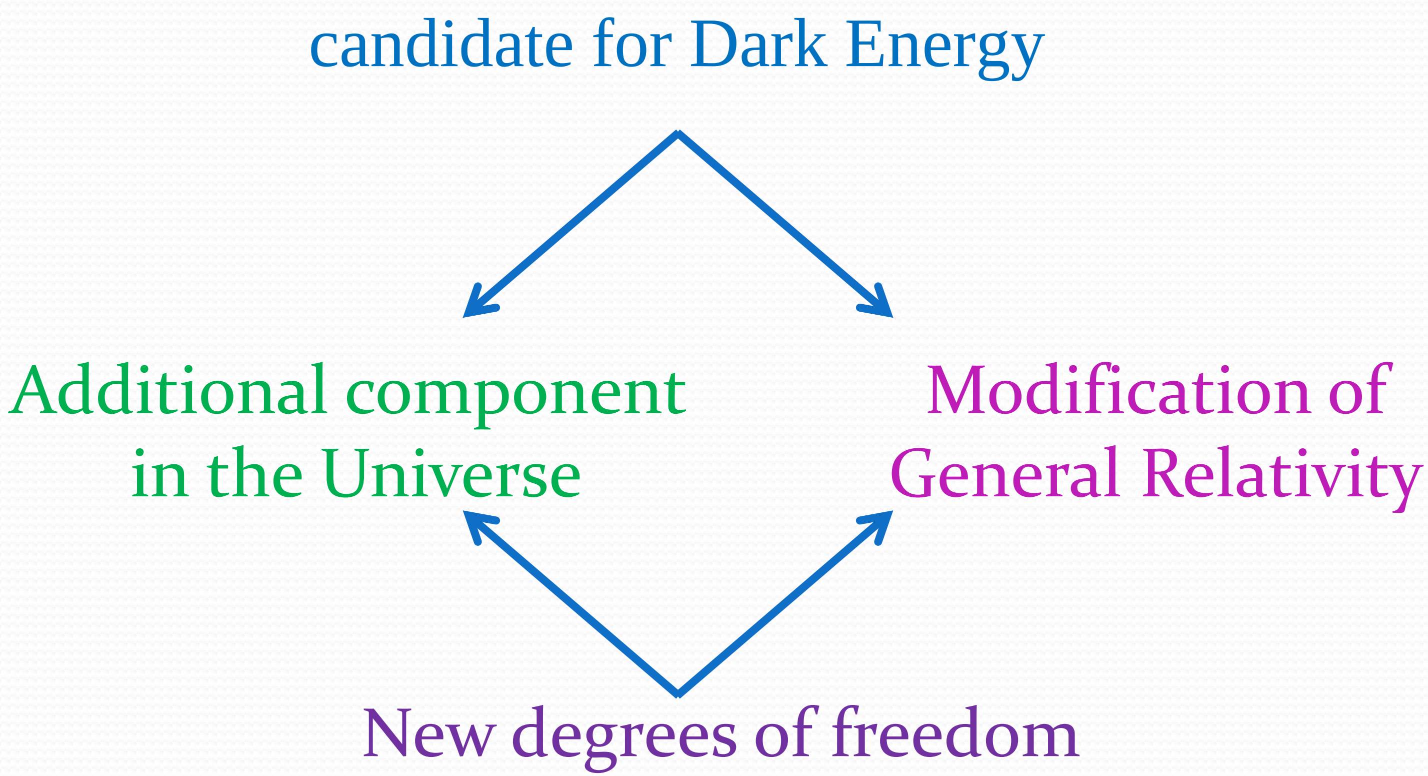
candidate for Dark Energy

Additional component in the Universe

Modification of General Relativity

Tackling Dark Energy

- Either ignore the vacuum energy from particle physics and find another source for the acceleration



Tackling Dark Energy

- Either ignore the vacuum energy from particle physics and find another source for the acceleration

- Or try to reconciliate the amount of vacuum energy with the observed acceleration

Change natural scale

Find a way to "hide" a large vacuum energy

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Change natural scale

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Can we change Gravity to tackle this issue ?

Ghost-free Massive Gravity

$$\mathcal{U}_{\rm GF} = \left(\mathcal{K}^2_{\mu\nu} - \mathcal{K}^2\right) + \alpha_3\left(\mathcal{K}^3 + \cdots\right) + \alpha_4\left(\mathcal{K}^4 + \cdots\right)$$

- In 4d, there is a 2-parameter family of ghost free theories of massive gravity

$$\mathcal{K}^\mu_\nu[g,\eta] = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\eta_{\alpha\nu}}$$

CdR, Gabadadze, 1007.0443
CdR, Gabadadze, Tolley, 1011.1232

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- Absence of ghost has now been proved fully non-perturbatively in many different languages

CdR, Gabadadze, 1007.0443
CdR, Gabadadze, Tolley, 1011.1232
Hassan & Rosen, 1106.3344
CdR, Gabadadze, Tolley, 1107.3820
CdR, Gabadadze, Tolley, 1108.4521
Hassan & Rosen, 1111.2070
Mirbabayi, 1112.1435
Hassan, Schmidt-May & von Strauss, 1203.5283
Deffayet, Mourad & Zahariade, 1207.6338

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- In 4d, there is a 2-parameter family of ghost free theories of massive gravity
- Absence of ghost has now been proved fully non-perturbatively in many different languages
- As well as around *any reference metric,* be it dynamical or not → **BiGravity !!!**

Hassan, Rosen & Schmidt-May, 1109.3230
Hassan & Rosen, 1109.3515

Degrees of Freedom

Massive Gravity

- 1 massive spin-2
  - 2 helicity-2
  - 2 helicity-1
  - 1 helicity-0

5 dofs

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5 dofs

- 2 dof in metric (after gauge fixing)
- 3 Stückelberg fields

Restore diff invariance

$$\sqrt{g^{\mu\alpha}\eta_{\alpha\nu}}$$

$$\eta_{ab}\partial_\mu\phi^a\partial_\nu\phi^b$$

$$\sqrt{g^{\mu\alpha} f_{\alpha\nu}}$$

$$f_{ab}\partial_\mu \phi^a \partial_\nu \phi^b$$

Degrees of Freedom

Massive Gravity

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5 dofs

- 2 dof in metric (after gauge fixing)
- 3 Stückelberg fields

Restore diff invariance

Bi-Gravity

- 1 massive & 1 massless spin-2
  - 2x2 helicity-2
  - 2 helicity-1
  - 1 helicity-0

7 dofs

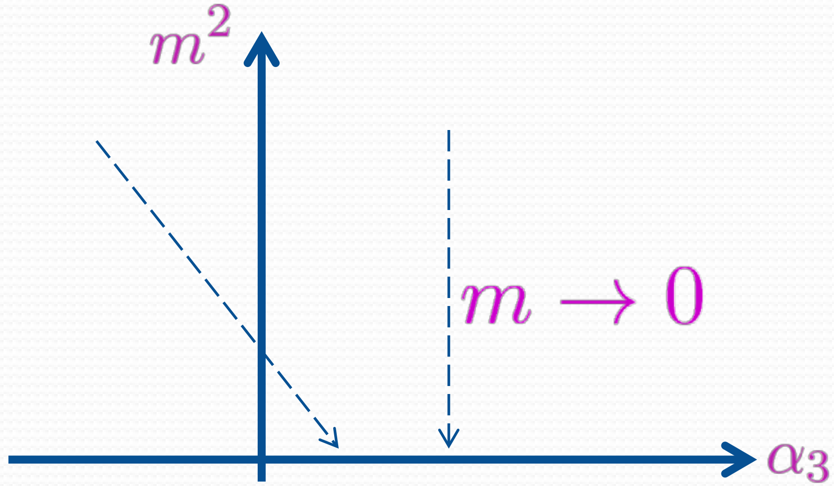
- 2x2 dof in both metrics (after gauge fixing)
- 3 Stückelberg fields

Restore 2nd copy of diff invariance

Massless limit

- In the massless limit, the helicity-0 mode still couples to matter $\pi\ T$

- Massless limit is *smooth* thanks to **Vainshtein Mechanism** (helicity-0 mode decouples)



Massless limit

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- Massless limit is *smooth* thanks to **Vainshtein Mechanism** (helicity-0 mode decouples)
- If gravity couples to matter in a diff. invariant way

$$h_{\mu\nu}T^{\mu\nu} \quad \text{with} \quad \partial_\mu T^{\mu\nu}=0$$

But…

But…

- The Vainshtein mechanism always comes hand in hand with ***superluminalities***…

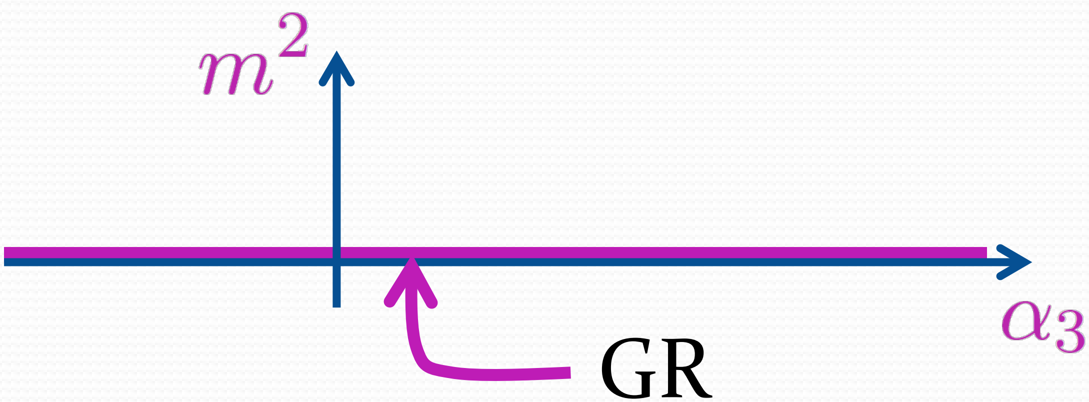
This doesn’t necessarily mean CTCs,
but - there is a family of preferred frames
- there is no absolute notion of light-cone.

Burrage, CdR, Heisenberg & Tolley, 1111.5549

But…

- The Vainshtein mechanism always comes hand in hand with ***superluminalities***…

- Most bounds on the graviton mass are really bounds on the helicity-0 mode.



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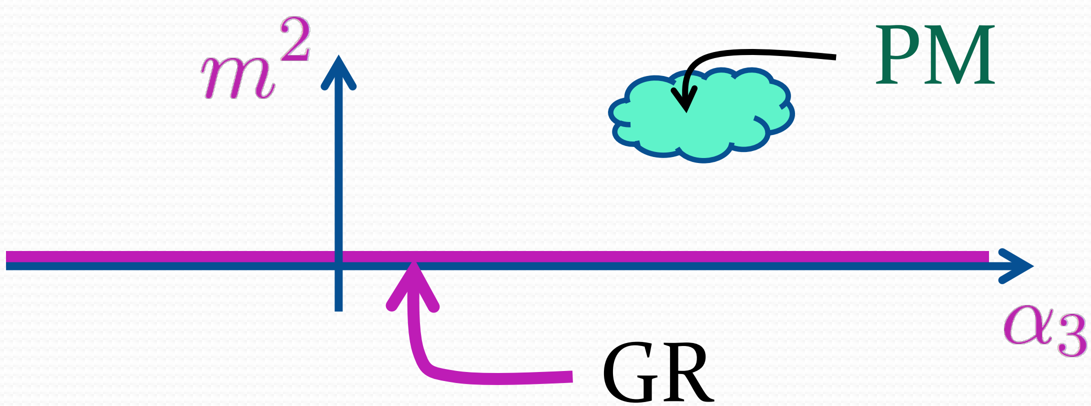
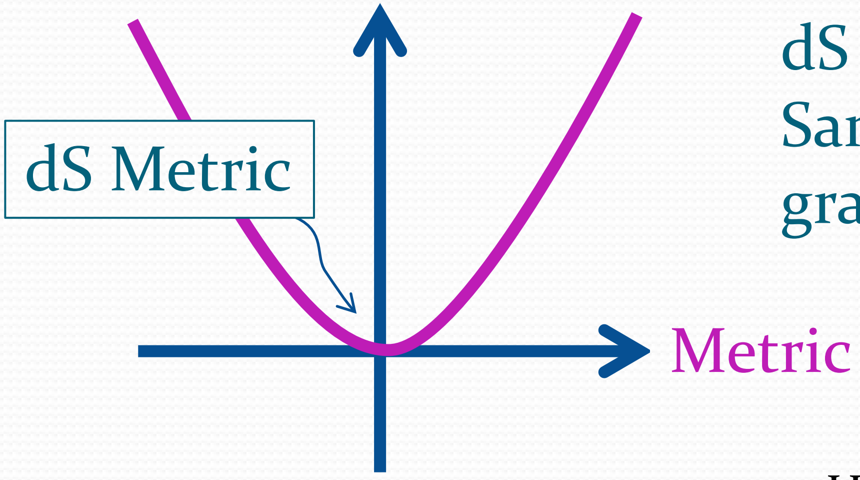


Is there a different region in parameter space where the helicity-0 mode could also be absent ???

Change of Ref. metric

$$S = \int \sqrt{-g}\frac{M_{\rm Pl}^2}{2}\left(R - \text{Mass Term}\right)$$

- Consider massive gravity around dS as a ***reference*** !



dS is still a maximally symmetric ST
Same amount of symmetry as massive gravity around Minkowski !

Hassan & Rosen, 2011

Massive Gravity in de Sitter

- Only the helicity-o mode gets 'seriously' affected by the dS reference metric

$$-\frac{m^4}{2}(\partial\pi)^2 \longrightarrow -\frac{m^2}{2}\left(m^2-(d-2)H^2\right)(\partial\pi)^2$$

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Higuchi, NPB**282**, 397 (1987)

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$$m^2 < (d-2)H^2 \longrightarrow$$

Deser & Waldron, hep-th/0103255
Grisa & Sorbo, 0905.3391
Fasiello & Tolley, 1206.3852

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Deser & Waldron, 2001

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Recover a symmetry $h_{\mu\nu} \to h_{\mu\nu} + D_\mu D_\nu \xi - (d-2)H^2 \xi \gamma_{\mu\nu}$

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- Is different from *FRW models* where the kinetic term disappears
  in this case the fundamental theory has a helicity-0 mode but it cancels on a specific background
- Is different from *Lorentz violating MG*
  no Lorentz symmetry around dS, but still have same amount of symmetry.

(Partially) massless limit

- Massless limit

GR + mass term

$$m \to 0$$

Recover 4d diff invariance

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$$

GR

in 4d: 2 dof (helicity 2)

Deser & Waldron, 2001

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- Partially Massless limit

GR + mass term

$m^2 \to (d-2)H^2$

Recover 1 symmetry

$$h_{\mu\nu} \to h_{\mu\nu} + D_\mu D_\nu \xi - (d-2)H^2 \xi \gamma_{\mu\nu}$$

Massive GR

4 dof (helicity 2 &1)

Implications of the Symmetry

- GR

  4d diff invariance

  $$h_{\mu\nu} \to h_{\mu\nu} + \partial_{(\mu}\xi_{\nu)}$$

  -Kills 3 dofs
  -Imposes matter to be conserved !

  $$D_\mu T^{\mu\nu} = 0$$

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new symmetry

$$h_{\mu\nu} \to h_{\mu\nu} + D_\mu D_\nu \xi - (d-2)H^2 \xi \gamma_{\mu\nu}$$

-Kills 1 dof
-Does NOT impose matter to be conserved !

$$D_\mu T^{\mu\nu} = \mathcal{O}(m^2)$$

Deser & Waldron

Implications of the Symmetry

- GR

4d diff invariance

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-Kills 1 dof
-Does NOT impose matter to be conserved !
- Instead

$$D_\mu D_\nu T^{\mu\nu} \sim m^2 T$$

PM symmetry fixes the vacuum energy to 0 !
First symmetry which could explain the CC problem !

Non-linear partially massless

Non-linear partially massless

- Let's start with ghost-free theory of MG,

$$\mathcal{U}_{\rm GF} = \left(\mathcal{K}^2_{\mu\nu} - \mathcal{K}^2\right) + \alpha_3 \left(\mathcal{K}^3 + \cdots\right) + \alpha_4 \left(\mathcal{K}^4 + \cdots\right)$$

- But around dS $\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\gamma_{\alpha\nu}}$

$\gamma_{\alpha\nu}$ ↳ dS ref metric

Non-linear partially massless

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- But around dS $\mathcal{K}^\mu_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\gamma_{\alpha\nu}}$

  dS ref metric

- And derive the 'decoupling limit'
  (*ie* leading interactions for the helicity-0 mode)

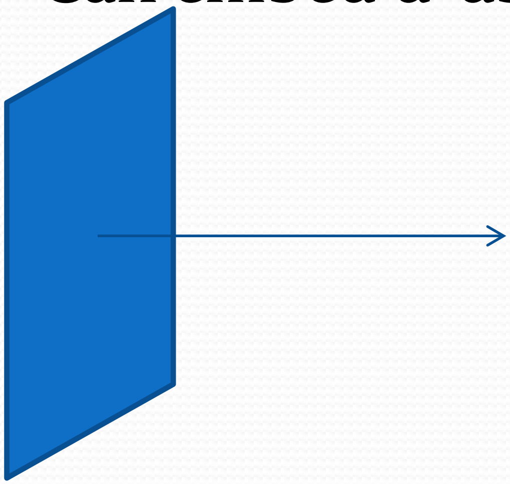
But we need to properly identify the helicity-0 mode first….

CdR & Sébastien Renaux-Petel, arXiv:1206.3482

Helicity-0 on dS

- To identify the helicity-0 mode *on de Sitter*, we copy the procedure *on Minkowski*.

- Can embed d-dS into (d+1)-Minkowski:



$$
\begin{aligned}
\mathrm{d}s^2 &= \mathrm{d}y^2 + e^{-2Hy}\gamma^{(\mathrm{dS})}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu \\
&= \eta_{AB}\mathrm{d}Z^A \mathrm{d}Z^B
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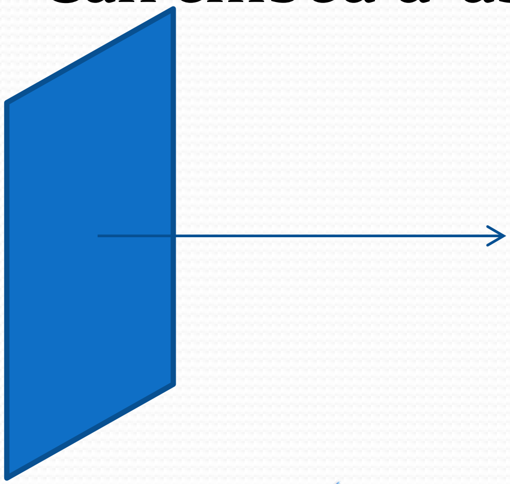

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\end{aligned}
$$

$$
\begin{aligned}
\tilde{\gamma}_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu &= \left(\tilde{\eta}_{AB}\mathrm{d}Z^A\mathrm{d}Z^B\right)|_{\mathrm{projected}} = \left(\eta_{MN}\partial_A\phi^M\partial_B\phi^N\mathrm{d}Z^A\mathrm{d}Z^B\right)|_{\mathrm{projected}} \\
&= \gamma_{MN}\partial_\mu\phi^M\partial_\nu\phi^N\mathrm{d}x^\mu\mathrm{d}x^\nu .
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CdR & Sébastien Renaux-Petel, arXiv:1206.3482

Helicity-0 on dS

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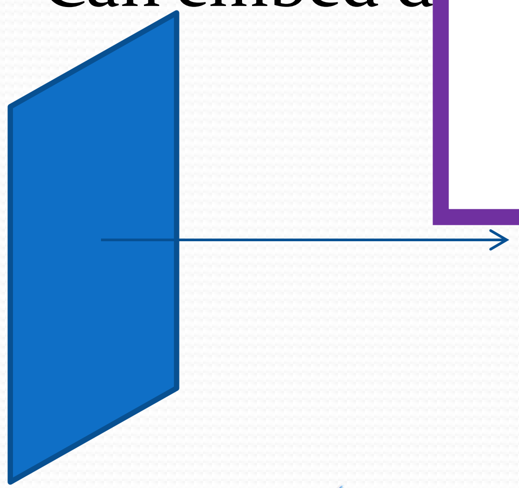
- Can embed d

$$\phi^A = Z^a - \partial^A \pi$$



π behaves as a scalar in the dec. limit and captures the physics of the helicity-o mode

$$\tilde{\gamma}_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu = \left(\tilde{\eta}_{AB} \mathrm{d}Z^A \mathrm{d}Z^B\right)|_{\mathrm{projecte}}$$
$$= \gamma_{MN} \partial_\mu \phi^M \partial_\nu \phi^N \mathrm{d}x^\mu \mathrm{d}x^\nu .$$

CdR & Sébastien Renaux-Petel, arXiv:1206.3482

Helicity-0 on dS

- To identify the helicity-0 mode *on de Sitter*, we copy the procedure *on Minkowski*.

- The covariantized metric fluctuation is expressed in terms of the helicity-0 mode as

$$
\begin{aligned}
H_{\mu\nu} &= h_{\mu\nu} + 2\Pi_{\mu\nu} - \Pi^2_{\mu\nu} \\
&+ H^2 \left((\partial\pi)^2 (\gamma_{\mu\nu} - 2\Pi_{\mu\nu}) - D^\alpha \pi D^\beta \pi \Pi_{\mu\alpha} \Pi_{\nu\beta} \right) + \mathcal{O}(H^4)
\end{aligned}
$$

$$\Pi_{\mu\nu} = D_\mu D_\nu \pi$$

in any dimensions…

CdR & Sébastien Renaux-Petel, arXiv:1206.3482

Decoupling limit on dS

- Using the properly identified helicity-0 mode, we can derive the decoupling limit on dS
- Since we need to satisfy the Higuchi bound,

$$M_{\rm Pl} \to \infty \quad m \to 0 \quad H \to 0$$

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- The resulting DL resembles that in Minkowski (Galileons), but with specific coefficients…

CdR & Sébastien Renaux-Petel, arXiv:1206.3482

DL on dS

$(\partial\pi)^2(\partial^2\pi)^{n-2}$

$$\mathcal{L}_\pi^{(\mathrm{dec})} = \underbrace{\sum_n c_n(H^2)\ \mathcal{L}_{\mathrm{Gal}}^{(n)}}_{\text{d terms}} + \underbrace{\text{non-diagonalizable terms mixing h and } \pi.}_{\text{d-3 terms}}$$

d terms + d-3 terms

(d-1) free parameters (m^2 and $\alpha_3,\ldots,d$)

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d terms + d-3 terms

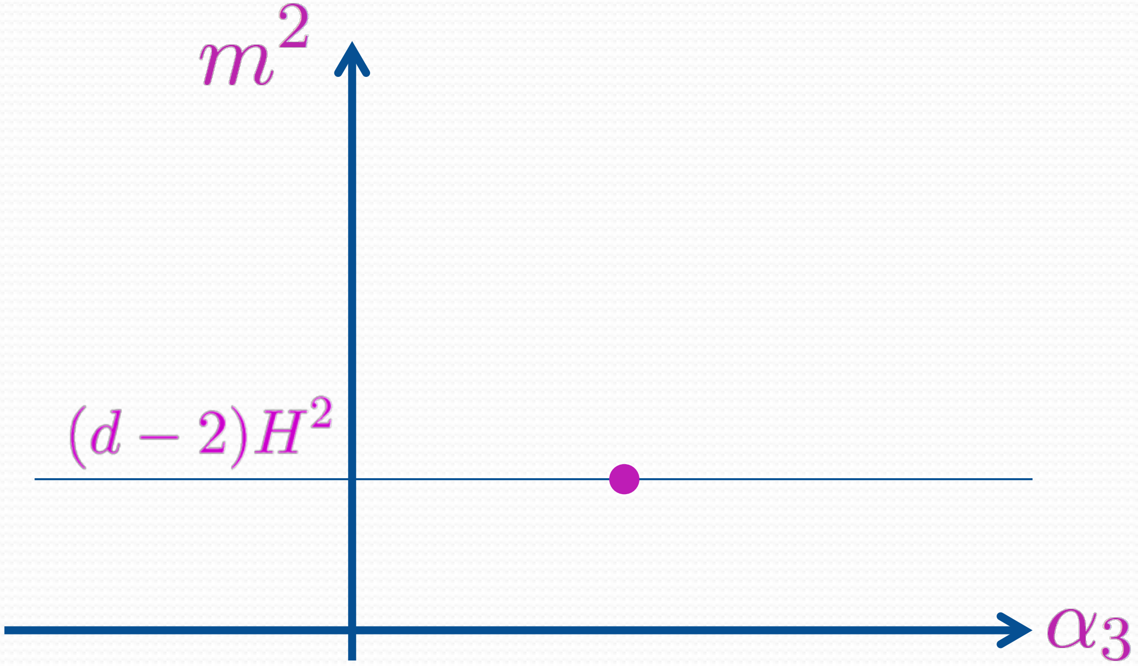
(d-1) free parameters (m^2 and $\alpha_{3,\dots,d}$)

- The kinetic term vanishes if $m^2 = (d-2)H^2$
- All the other interactions vanish simultaneously if

$$\alpha_3 = -\frac{1}{3}\frac{d-1}{d-2}$$

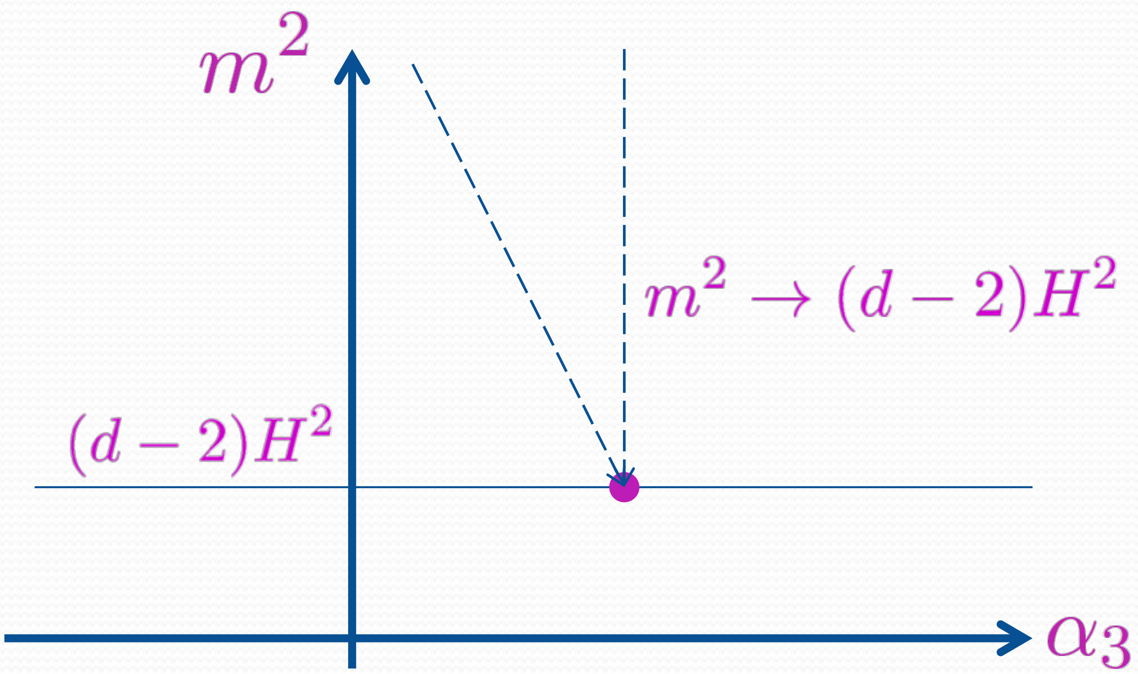
$$\alpha_{n-1} + n\,\alpha_n = 0 \quad \forall\, n > 3$$

Partially massless limit



Coupling to matter π $\mathcal{T}$ eg. $\mathcal{T} = m^2 T - \partial_\mu \partial_\nu T^{\mu\nu} = 0$

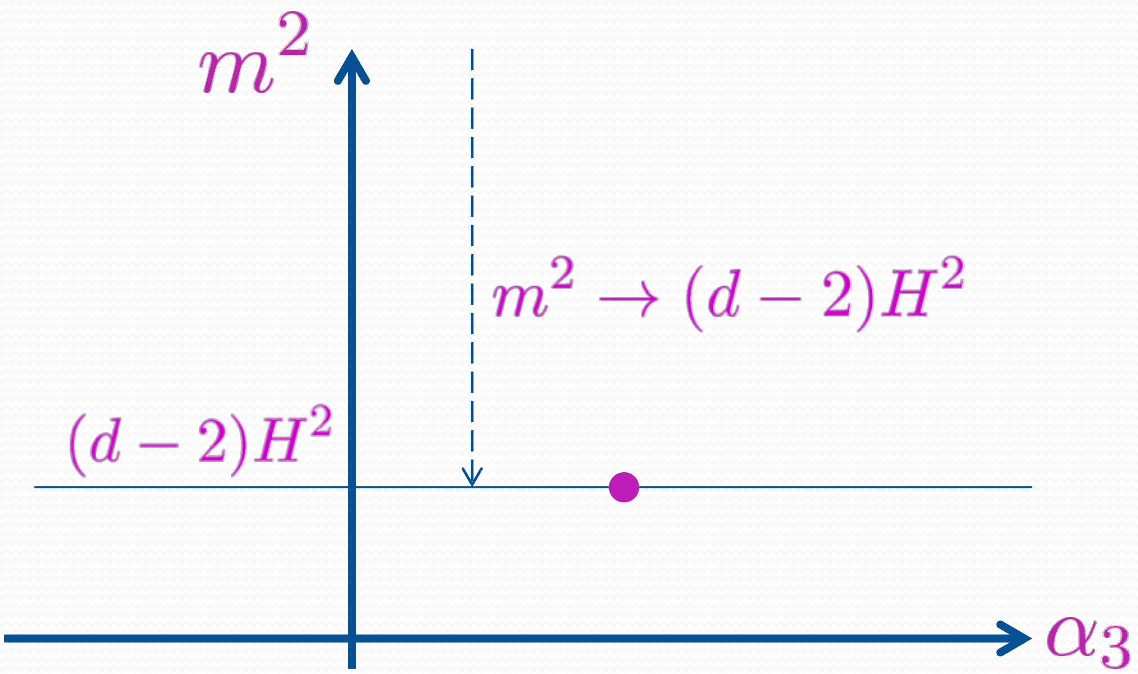
Partially massless limit



The symmetry cancels the coupling to matter

There is no Vainshtein mechanism, but there is no vDVZ discontinuity…

Partially massless limit



Unless we take the limit $m^2 \to (d-2)H^2$ without considering the PM parameters α.

In this case the standard Vainshtein mechanism applies.

Partially massless

- We **uniquely** identify the non-linear candidate for the Partially Massless theory to all orders.
- In the DL, the helicity-0 mode entirely disappear in any dimensions
- What happens beyond the DL is still to be worked out

See Deser&Waldron
Zinoviev

- As well as the non-linear realization of the symmetry…

Work in progress with Kurt Hinterbichler, Rachel Rosen & Andrew Tolley