Scale Dependence of Non-Gaussian Statistics

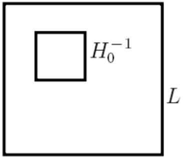
Elliot Nelson

Institute for Gravitation and the Cosmos
Pennsylvania State University

Workshop on Cosmic Acceleration, 2012
Carnegie Mellon University

Introduction

- Inflation:
  - Model-dependent predictions for non-Gaussian inhomogeneities in a large, inflating volume:
    $\delta T_{\mu\nu},\ \delta g_{\mu\nu} \Rightarrow \zeta(x) \Rightarrow (\delta T/T)_{\rm CMB},\ \delta\rho/\rho$
  - The observable universe as a small patch
- *What is the relationship between the non-Gaussian statistics in a large volume to non-Gaussian statistics observed in a small subsample region?*
  How are short modes correlated to long modes?
- *Statistical problem.* Assume only:
  - Observable universe is a small patch
  - Initial field of small fluctuations



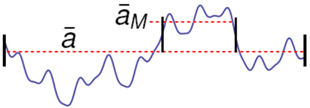
Primordial Curvature Perturbation

The primordial curvature perturbation ζ

- Fluctuations in the scale factor from inflation: $ds^2 = -dt^2 + \bar{a}(t)^2(1 + 2\zeta(\vec{x}, t))d\vec{x}^2$
- ζ at end of inflation is a static, fixed field; directly related to observables

$\zeta(x)$ in two boxes:

$$\begin{aligned} a(x) &= \bar{a}(1 + \zeta(x)) \,, \quad x \in Vol_L \\ &= \bar{a}_M(1 + \tilde{\zeta}(x)) \,, \quad x \in Vol_M, \end{aligned}$$



with $\bar{a}_M = \bar{a}(1 + \bar{\zeta})$.

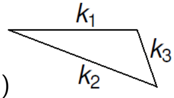
- Break ζ into long and short-wavelength modes:

$$\zeta(x) = \left[\int_{L^{-1}}^{M^{-1}} + \int_{M^{-1}}^{R^{-1}} \right] d^3k \zeta_{\mathbf{k}} e^{i\mathbf{kx}} \equiv \bar{\zeta} + \zeta_s$$

Non-Gaussianity

Interactions in early universe $\Rightarrow$ non-Gaussian statistics

- *Non-Gaussianity correlated modes*
- n-point functions of $\zeta_{\mathbf{k}}$:
  - $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle \equiv (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2) P_\zeta(k_1)$,
    $\mathcal{P}_\zeta(k) \equiv \frac{k^3}{2\pi^2} P_\zeta(k)$, $\langle \zeta(x)^2 \rangle = \int \frac{dk}{k} \mathcal{P}_\zeta(k)$
  - $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle_C \equiv (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\zeta(k_1, k_2, k_3)$
  - $\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \times \ldots \zeta_{\mathbf{k}_n} \rangle_C$: more information



Goal: Compare n-point functions in large and small box

- Gaussian field: no correlation to background, same statistics on all scales
- Local ansatz: ζ as local nonlinear transformation of Gaussian field
  - $\zeta = \sigma + f_{NL}(\sigma^2 - \langle \sigma^2 \rangle)$
  - $\mathcal{P}_\zeta = \mathcal{P}_\sigma + O(f_{NL}^2 \mathcal{P}_\sigma^2)$

Local Ansatz

- Split Gaussian field into long/short wavelengths, $\sigma = \bar{\sigma} + \sigma_s$:

$$\begin{aligned}\zeta(x) &= \sigma + f_{NL}(\sigma^2 - \langle\sigma^2\rangle) \\ &= \underbrace{\bar{\sigma} + f_{NL}(\bar{\sigma}^2 - \langle\bar{\sigma}^2\rangle)}_{\bar{\zeta}} + \underbrace{(1 + 2f_{NL}\bar{\sigma})\sigma_s + f_{NL}(\sigma_s^2 - \langle\sigma_s^2\rangle)}_{\zeta_s}\end{aligned}$$

- Local model applies in the small box:

$$\tilde{\zeta} = \tilde{\sigma} + \tilde{f}_{NL}(\tilde{\sigma}^2 - \langle\tilde{\sigma}^2\rangle)$$

- Change in "level of non-Gaussianity" $f_{NL}\mathcal{P}^{1/2}$:

$$\boxed{\tilde{f}_{NL}\tilde{\mathcal{P}}^{1/2} = f_{NL}\mathcal{P}^{1/2}(1 + 2f_{NL}\bar{\sigma})^{-1}}$$

- Dimensionless moments keep same scaling:

$$\mathcal{M}_n \equiv \frac{\langle\zeta^n\rangle_c}{\langle\zeta^2\rangle^{n/2}} \propto (f_{NL}\mathcal{P}^{1/2})^{n-2} \simeq \tilde{\mathcal{M}}_n$$

$\Rightarrow$ *Local ansatz with natural series preserved on small scales*

Observer's View

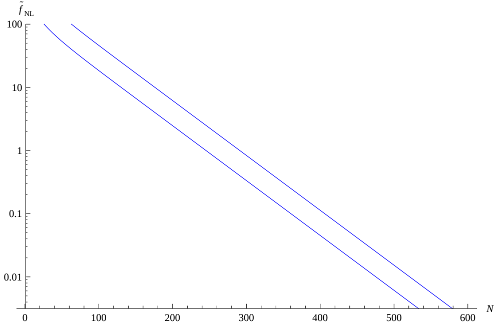


Figure: $\tilde{f}_{NL}\tilde{\mathcal{P}}^{1/2}/f_{NL}\mathcal{P}^{1/2} = 0.8, 0.5$ for $\bar{\zeta}/\langle\bar{\zeta}^2\rangle^{1/2} = 3$.

- WMAP7: $\tilde{f}_{NL}^{\text{local}} = 32 \pm 21$ (1σ), Planck: ± 5
- We assumed $\mathcal{P}(k) = \mathcal{P}_0 k^{n_s - 1}$ has spectral index $n_s = 0.96$ on all scales

Longer inflation $\Rightarrow$ more background modes $\Rightarrow$ uncertainty in measuring quantities in large volume

More General Local Models

Fine-tuned Local Ansatz:

$$\zeta = \sigma + N_p \sigma^p + N_{p+1} \sigma^{p+1} + \ldots$$

- Moments may be out of order in large box, $\mathcal{M}_{n+1} \nless \mathcal{M}_n$:
- In small box, regenerate lower terms, naturally ordered series:

$$\tilde{\mathcal{M}}_n \propto C(f_{NL}^{eff} \tilde{\mathcal{P}}^{1/2})^{n-2} = (N_p \bar{\sigma}^{p-1}) \left(\frac{1 + \bar{\zeta}}{\bar{\sigma}} \tilde{\mathcal{P}}^{1/2} \right)^{n-2} \quad (n \leq p)$$

Highly Non-Gaussian Ansatz:

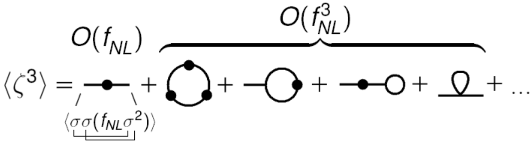
$$\zeta = \sigma^p + N_{p+1} \sigma^{p+1} + \ldots$$

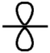
- Regenerate all lower order terms;
  Dominant linear term in sufficiently small subsamples:

$$\tilde{\mathcal{M}}_n \propto (f_{NL}^{eff} \tilde{\mathcal{P}}^{1/2})^{n-2} = \left(\frac{1}{\bar{\sigma}} \mathcal{P}^{1/2} \right)^{n-2}$$

$\Rightarrow$ *WNG Local ansatz with hierarchical scaling is statistically natural*

Bispectrum Shapes for Local Ansatz

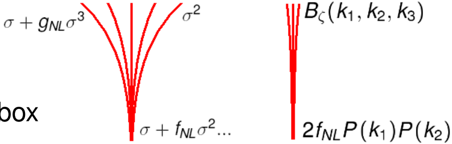


- For $\zeta = \sigma + f_{NL}\sigma^2$,
  $B_\zeta(k_1, k_2, k_3) = 2f_{NL}P_\sigma(k_1)P_\sigma(k_2) + \text{cyc.} \sim$ 
- For $\zeta = \sigma + N_p\sigma^p + \ldots$,
  $B_\zeta(k_1, k_2, k_3) \simeq N_p\langle\sigma_{\mathbf{k}_1}\sigma_{\mathbf{k}_2}(\sigma^p)_{\mathbf{k}_3}\rangle \sim$  $(p = 6) \propto$ 
- For $\zeta = \sigma^p + \ldots$,
  $B_\zeta(k_1, k_2, k_3) \simeq \langle(\sigma^p)_{\mathbf{k}_1}(\sigma^p)_{\mathbf{k}_2}(\sigma^p)_{\mathbf{k}_3}\rangle \sim$  $(p = 2) \sim ($  $) \times \mathcal{P}_\sigma \ln(k_{\min}L)$
- Regenerate quadratic term in small box $\Rightarrow$  is dominant piece

Higher n-point functions behave similarly
$\Rightarrow$ *Shapes of $\langle\zeta^n\rangle$ for local ansatz change only logarithmically with scale, are protected from arbitrary distortions from coupling to background*

[Bramante & Kumar 1107.5362]

Conclusion

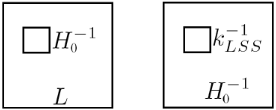


Key Points

- Non-Gaussian statistics in large box and subsamples can be different
  - On small scales, can recover nearly Gaussian field with hierarchical moments
  - Shapes of *n*-point functions are nearly the same on small scales

Future Work:

- Nonlocal models for ζ: scale-protected shapes?
  - Statistics from inflation
  - Bispectra to look for in LSS data
- Numerical simulations
- Inflation models: uncertainties in parameters from theory
- Large scale structure: new ways to search for NG
  - Local background effects on statistics to probe higher *n*-point functions, eg. halo bias



Thank You

EN, S. Shandera (arXiv:12xx.xxxx)