Galileons as Wess-Zumino Terms

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Based On

Based on work with Kurt Hinterbichler, Austin Joyce and Mark Trodden.

- Garrett Goon, Kurt Hinterbichler, Mark Trodden, "*General Embedded Brane Effective Field Theories*", arXiv:1103.6029v2 [hep-th]
- Garrett Goon, Kurt Hinterbichler, Mark Trodden, "*Symmetries for Galileons and DBI scalars on curved space*", arXiv:1103.5745v1 [hep-th]
- Garrett L. Goon, Kurt Hinterbichler, Austin Joyce, Mark Trodden, "*Galileons as Wess-Zumino Terms*", arXiv:1203.3191 [hep-th]

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- They exhibit Vainshtein screening mechanism and appear in modified gravity theories, ex. DGP and Massive Gravity.
- Galileons are the Goldstones of spacetime SSB in probe brane theories.
- This works treats Galileons using standard SSB methods, galileons require a higher dimensional construction and are Wess-Zumino terms.

Review of Galileons

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- In a certain limit, DGP effects are captured by a single scalar field,

$$S_{\mathrm{D.L.}} \sim \int d^4x \; -\frac{1}{2}(\partial\pi)^2 - \frac{a}{\Lambda^3}(\partial\pi)^2 \Box\pi + \frac{1}{M_4}\pi T + \ldots$$

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- In d-dimensions there are only $d+1$ Galileon terms, taking the form

$$\mathcal{L}_i \sim \pi(\partial^2\pi)^{i-1},\ i \in \{1, \ldots, d+1\}$$

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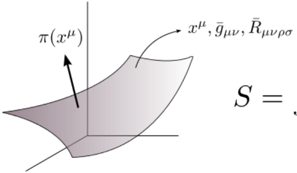
- Other terms in the EFT are higher order, $\sim (\partial\partial\pi)^n$.
- Galileons do not receive quantum corrections, $\delta\Gamma \sim (\partial\partial\pi)^n$. Proof is diagrammatic.

Geometric Picture of Galileons

Galileons as Goldstones

- Galileons arise from probe brane scenarios using Lovelock invariants in the action (de Rham et al., 1003.5917).



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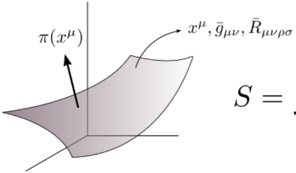

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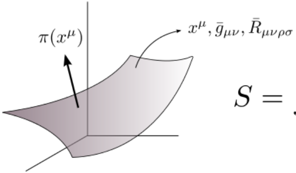

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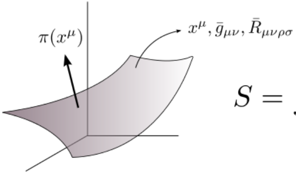

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- Other Galileon theories come from brane perspective (Goon et al., 1103.5745),(Goon et al., 1003.5917).
- Galileon type theories describe Goldstones of spacetime SSB.

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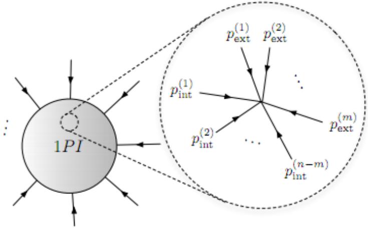
- New perspectives may shed light on non-renormalization theorem.



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- **Galileons are not strictly invariant.**

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- Something's missing.

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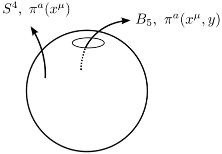

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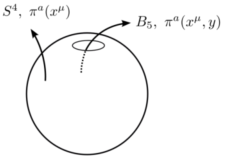

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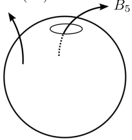

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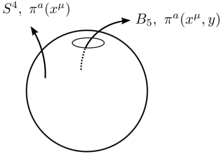

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- Global considerations show that α is quantized, not renormalized.

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- No obvious non-renormalization theorem.

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- Further investigate quantum aspects of galileon type theories.

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