A new way to count degrees of freedom in ghost free massive gravity

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Outline

- dRGT massive gravity
- Vierbein formulation
- Counting degrees of freedom
  - Lorentz symmetry breaking
  - Diffeomorphism invariance breaking
  - Additional constraint
- Summary

Massive gravity theories

- At the quadratic order, we have Fierz-Pauli theory (5 d.o.f.)

$$S_{mass} \propto m^2 \int d^4x \left(h_{\mu\nu} h^{\mu\nu} - h^2 \right)$$

➢ NOT in agreement with solar system tests

- Goal: finding consistent non-linear completions of Fierz-Pauli theory that allow for the recovery of GR at solar system scales (via the Vainshtein mechanism)

- Boulware and Deser (1972): presence of a ghostlike 6^{th} degree of freedom (BD ghost)

- Is it possible to find a non-linear theory of massive gravity devoid of the BD ghost?

dRGT massive gravity

- Mass term constructed with two metrics
  - a dynamical one g
  - a background metric f (non-dynamical) usually taken to be flat

- de Rham, Gabadadze, Tolley (2010-2011): ghost free massive gravity

$$S_{dRGT} \equiv \frac{1}{2}\int d^4x\sqrt{-g}R + \sum_{n=0}^{3}\alpha_n \int d^4x\sqrt{-g}E_n\left(\sqrt{g^{-1}f}\right)$$

- 3 parameter family of non-trivial theories

$$E_0(X) = 1$$
$$E_1(X) = \mathrm{Tr}(X)$$
$$E_2(X) = \frac{1}{2}\left(\mathrm{Tr}\left(X^2\right) - \mathrm{Tr}(X)^2\right)$$
$$E_3(X) = \frac{1}{6}\left(\mathrm{Tr}(X)^3 - 3\mathrm{Tr}(X)\mathrm{Tr}\left(X^2\right) + \mathrm{Tr}\left(X^3\right)\right)$$

dRGT massive gravity

- Proven to be ghost free
  - at all orders in the decoupling limit dRGT (2010-2011)
  - fully nonlinearly in the Hamiltonian formalism Hassan, Rosen (2011)

- Lots of work has been done since
  - other independent proofs Kluson, Mirbabayi…
  - extension to bimetric theories Hassan, Rosen…
  - solutions Volkov, Mukohyama…
  - vielbein, multi-vielbein reformulation Hinterbichler, Rosen
  - …

Vierbein formulation of dRGT theory

- Reformulation of dRGT theory with vierbeins

  Hinterbichler, Rosen (2012)
- Starting point
  - E^A dynamical 1-form
  - $L^A = dx^A$ non-dynamical "background" 1-form

- Mass terms proportional to

$$\epsilon_{A_1 A_2 A_3 A_4} E^{A_1} \wedge E^{A_2} \wedge E^{A_3} \wedge E^{A_4} \quad \text{(just a cosmological constant)}$$

$$\epsilon_{A_1 A_2 A_3 A_4} L^{A_1} \wedge E^{A_2} \wedge E^{A_3} \wedge E^{A_4}$$

$$\epsilon_{A_1 A_2 A_3 A_4} L^{A_1} \wedge L^{A_2} \wedge E^{A_3} \wedge E^{A_4}$$

$$\epsilon_{A_1 A_2 A_3 A_4} L^{A_1} \wedge L^{A_2} \wedge L^{A_3} \wedge E^{A_4}$$

➢ 16 degrees of freedom: how do we get to 5?

Vierbein formulation of dRGT theory

- The vierbein action

$$S_{dRGT} \equiv \frac{1}{2}\int \Omega^{AB} \wedge E^*_{AB} + \sum_{n=0}^{3} \beta_n \int L^{A_1} \wedge \ldots \wedge L^{A_n} \wedge E^*_{A_1 \ldots A_n}$$

where $E^*_{A_1 \ldots A_n} \equiv \frac{1}{(4-n)!} \epsilon_{A_1 \ldots A_n \ldots A_4} E^{A_{n+1}} \wedge \ldots \wedge E^{A_4}$

$$\Omega^{AB} \equiv d\omega^{AB} + \omega^{A}{}_{C} \wedge \omega^{CB} \qquad (dE^A + \omega^{A}{}_{B} \wedge E^B = 0)$$

- Equations of motion

$$G_A = t_A$$
$$G_{AB} = t_{AB}$$

where

$$G_A \equiv -\frac{1}{2}\Omega^{BC} \wedge E^*_{ABC} \equiv G_A{}^{B} E^*_B \quad \text{(Einstein 3-form)}$$

$$t_A \equiv \sum_{n=0}^{3} \beta_n L^{A_1} \wedge \ldots \wedge L^{A_n} \wedge E^*_{AA_1 \ldots A_n} \equiv t_A{}^{B} E^*_B \quad \text{(energy-momentum 3-form)}$$

Constraints arising from local Lorentz symmetry breaking

- The kinetic term is invariant under local Lorentz transformations

$$G_{[AB]} = 0$$

- 6 constraints arising from the breaking of local Lorentz invariance by the mass term

$$t_{[AB]} = 0$$

- This naïvely eliminates 6 degrees of freedom: 16-6=10

- In some cases this implies a "symmetry" condition on the 1-forms E^A and their dual vectors e_A:

$$e_A{}^{\mu} L_{B\mu} = e_B{}^{\mu} L_{A\mu} \quad \Leftrightarrow \quad e^{AB} \text{ symmetric}$$

$$\beta_0 \neq 0, \quad \beta_1 \neq 0$$
$$\beta_0 \neq 0, \quad \beta_3 \neq 0$$

Symmetric vierbeins and matrix square roots

- **Digression:** "symmetric" vierbein condition necessary to show the equivalence between the metric and vierbein formulation
  - a real matrix doesn't always have a real square root
  - introducing "symmetric" vierbeins simplifies the problem

$$g_{\mu\nu} = \eta_{AB} E^A{}_\mu E^B{}_\nu, \quad g^{\mu\nu} = \eta^{AB} e_A{}^\mu e_B{}^\nu$$

$$f_{\mu\nu} = \eta_{AB} L^A{}_\mu L^B{}_\nu, \quad f^{\mu\nu} = \eta^{AB} l_A{}^\mu l_B{}^\nu$$

$$e_A{}^\mu L_{B\mu} = e_B{}^\mu L_{A\mu}$$

$$\Rightarrow \left(\sqrt{g^{-1} f}\right)^\mu{}_\nu = e_A{}^\mu L^A{}_\nu$$

Symmetric vierbeins and matrix square roots

- Can the "symmetric" vierbein condition $e_A{}^\mu L_{B\mu} = e_B{}^\mu L_{A\mu}$ be imposed dynamically?
  - non-existence of a generalized polar decomposition for an invertible matrix

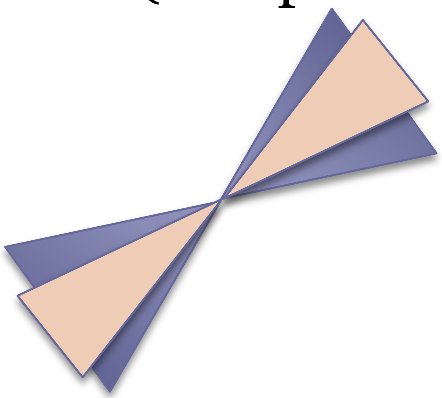
$$M = \Lambda . S$$

- Can it be imposed via local Lorentz transformation?
  - **Result:** the "symmetric" vierbein condition can be imposed via local Lorentz transformations **iff**
    1. the matrix $g^{-1}f$ admits a real square root γ and
    2. $f\gamma$ is symmetric

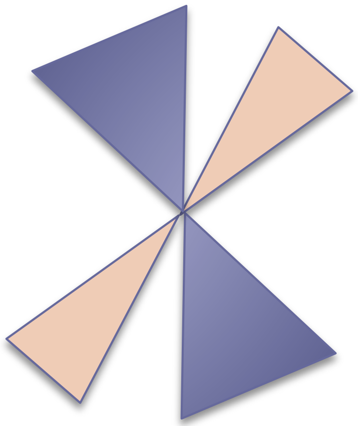
Symmetric vierbeins and matrix square roots

- What is the relationship between conditions i. and ii. ?

- If the light cones corresponding to the two metrics do not intersect (except at the origin)



$i. \Rightarrow ii.$



$i. \not\Leftarrow ii.$

➢ **One cannot impose the condition generically!**

Constraints arising from diffeomorphism invariance breaking

- Covariant derivative acting on forms

$$DF_A \equiv dF_A + \omega_A{}^B \wedge F_B$$

- The kinetic term is invariant under diffeomorphisms

$$DG_A = 0$$

- 4 constraints arising from the breaking of diffeomorphism invariance by the mass term

$$Dt_A = 0$$

- This eliminates 4 more degrees of freedom: 10-4=6

Additional constraint

- Naïve degree of freedom counting 16 – 6 – 4 = 6
- We need one more constraint in order to eliminate the 6th degree of freedom
- Particular case: $\beta_0 \neq 0, \quad \beta_1 \neq 0$
  - e^{AB} symmetric
  - Diffeomorphism invariance constraint $\omega^{BA}{}_{B} \equiv e_B{}^{\mu} \omega^{AB}{}_{\mu} = 0$
  - This eliminates the second order derivatives present in

$$E^A \wedge G_A = E^A \wedge t_A \quad \Rightarrow \quad \text{Scalar constraint (1}^{\text{st}}\text{order derivative eq.)}$$

- Same argument goes for $\beta_0 \neq 0, \quad \beta_2 \neq 0$ but not for $\beta_0 \neq 0, \quad \beta_3 \neq 0$

Pauli-Fierz limit

- We have 11 constraints, but do they really eliminate 11 degrees of freedom, or equivalently, are they independent?

- In the linearized limit, we exactly find the Fierz-Pauli constraints

$$\begin{aligned} Dt_A &= 0 \\ E^A \wedge G_A &= E^A \wedge t_A \end{aligned} \quad \Longrightarrow \quad \begin{aligned} \partial^\mu h_{\mu\nu} &= 0 \\ h &= 0 \end{aligned}$$

- Therefore our constraints are all independent

Summary

- Ghost free massive gravity theory formulation with vierbeins
  - Non-trivial relationship between exixtence of a real matrix square root of $g^{-1}f$ and the "symmetric" vierbein condition
- Covariant degree of freedom counting
- We find enough constraints for two of the three possible non-trivial mass terms
- It seems however that the third non-trivial case cannot be handled in the same manner

Thank you for your attention