Spatially Covariant Theories of a Transverse, Traceless Graviton

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with Justin Khoury & Andrew J. Tolley

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Cosmic Acceleration

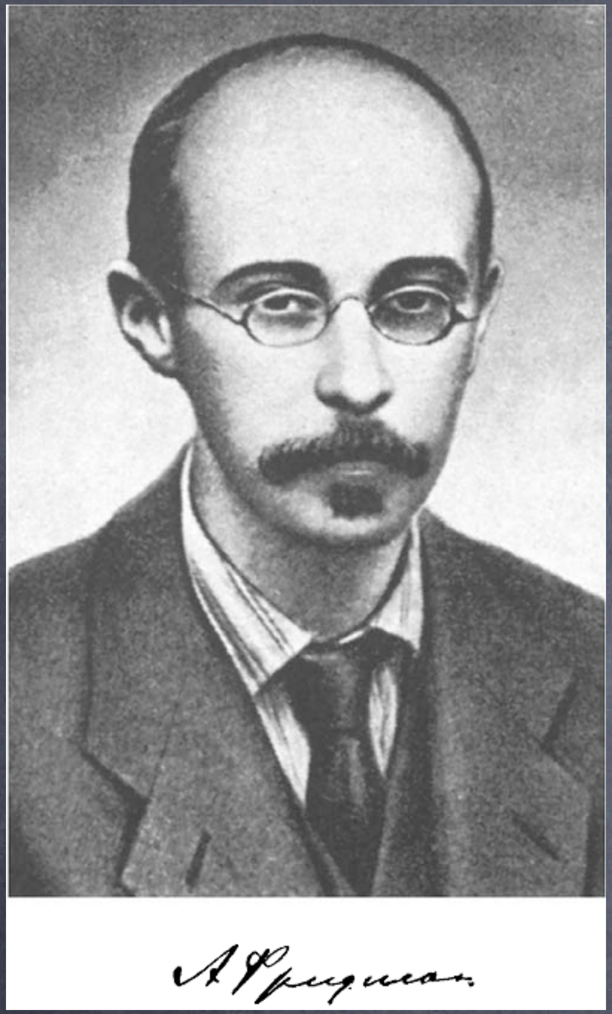
Saul Perlmutter, Brian P. Schmidt, Adam G. Riess

"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"

$$\ddot{a} > 0$$

2011

Implications of Acceleration



Alexander Friedmann

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

$$\ddot{a} > 0 \longrightarrow p < -\frac{1}{3}\rho$$

Cosmological Constant

$$S = \int \mathrm{d}^4x \sqrt{-g}\,\Lambda \longrightarrow p_\Lambda = -\rho_\Lambda$$

ΛCDM, the Concordance Model of Cosmology

$$\Omega_\Lambda \approx 0.7 \longrightarrow \rho_\Lambda \approx (meV)^4$$

Cosmological Constant Problem

Expectation	Apparent Reality
$\rho_\Lambda \approx M_{Pl}^4$	$\rho_\Lambda \approx (meV)^4 \approx 10^{-120} M_{Pl}^4$

- Cosmological Constant Problem - Why is ρ_Λ so small?
  - Cancellation of zero-point energies requires extreme fine-tuning
  - Weinberg's "No-Go" Theorem: Cannot relax dynamically to $\rho_\Lambda \approx 0$

S. Weinberg, Rev.Mod.Phys. 61 (1989) 1-23

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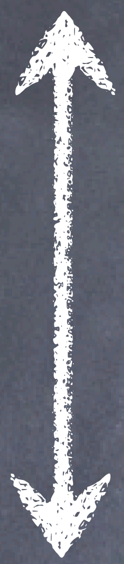
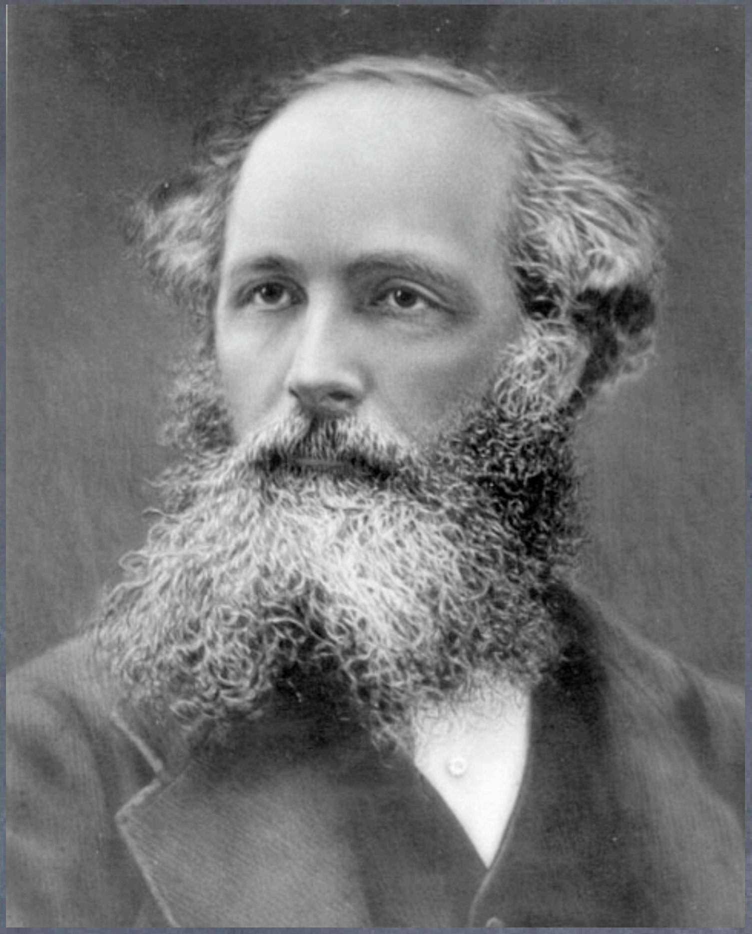
- Possible Solutions
  - Dark Energy
  - Modified Gravity
  - ?

Can General Relativity be modified without new degrees of freedom?

Degrees of Freedom

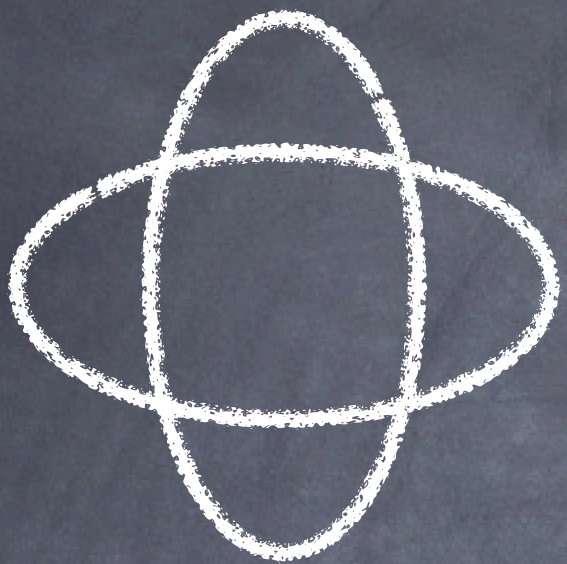
- Independent parameters needed to specify the state of a system
- Classical field theories have infinitely many degrees of freedom, but...
- Finitely many local degrees of freedom
- After quantization, these count particle polarization states

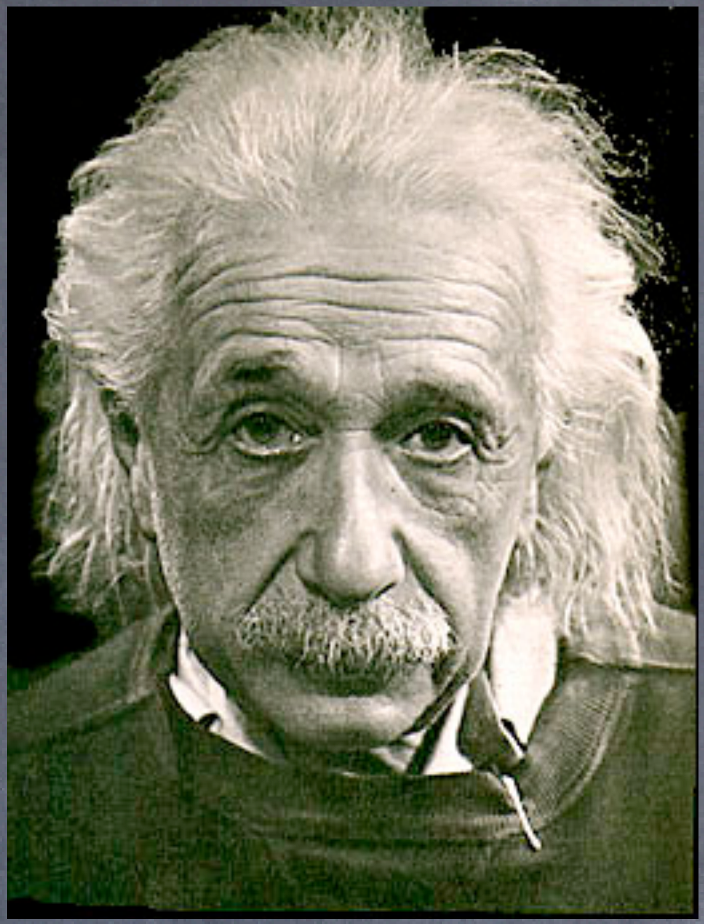




$$S = \int \mathrm{d}^4x \, F_{\mu\nu} F^{\mu\nu}$$

Field	Particle	Polarizations
A_μ	Photon	2





$$S = \int \mathrm{d}^4x \sqrt{-g}\, R$$

Field	Particle	Polarizations
$g_{\mu\nu}$	Graviton	2

Uniqueness Theorem

- In Lorentz covariant theories, polarization states are fixed by mass and spin of particles

- Weinberg's Theorem: GR is the unique Lorentz covariant theory of an interacting massless spin-2 particle

S. Weinberg
Phys. Rev. 138 (1965) B988-B1002

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→ Lorentz covariant modifications of GR introduce new degrees of freedom

Modifying the Graviton

- From binary pulsars, strong evidence for two polarizations
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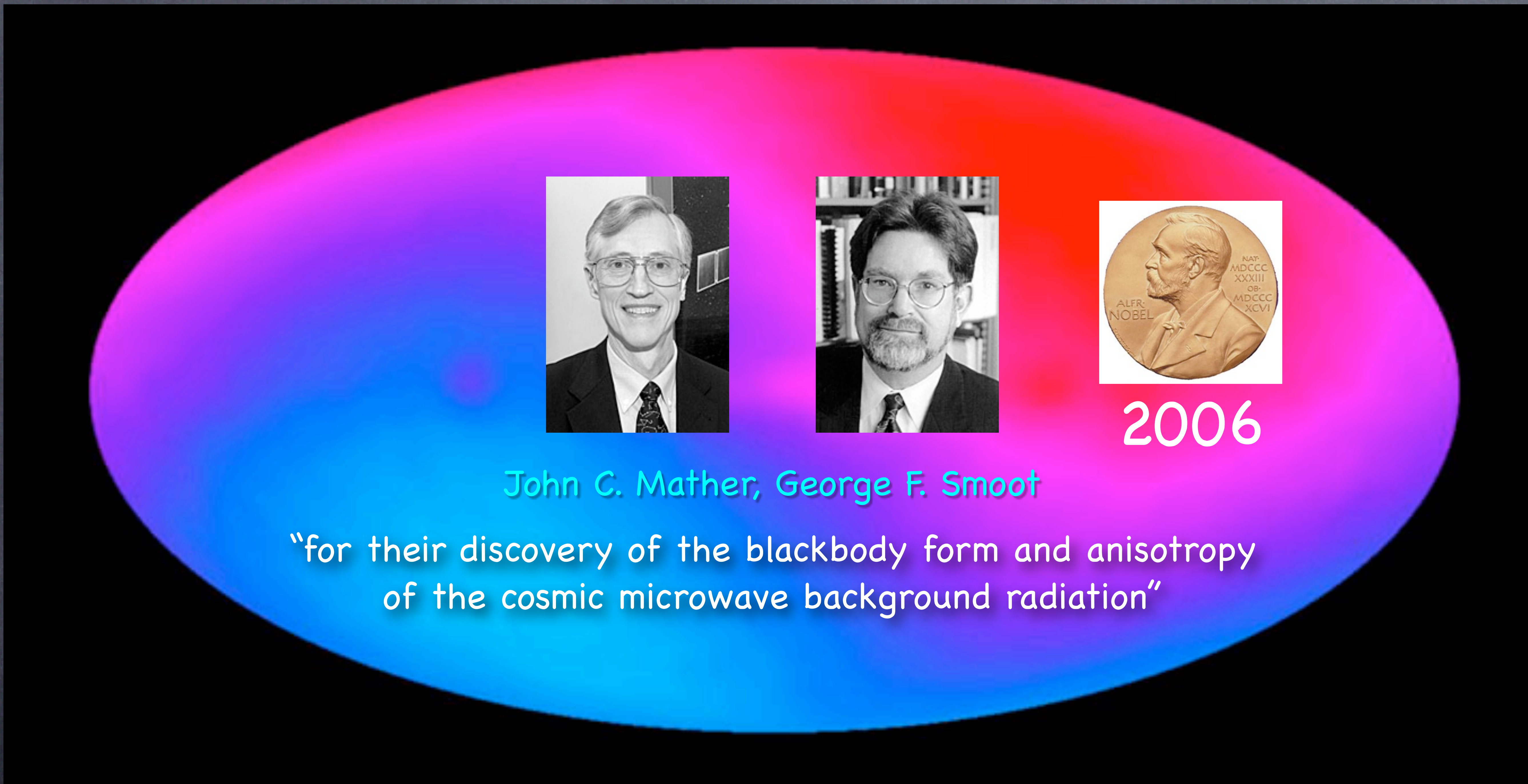
Break Lorentz covariance...

Explicitly!

Cosmic Rest Frame



2006

John C. Mather, George F. Smoot

“for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation”

Spontaneous or Explicit Symmetry Breaking?

ADM Action for GR

10		6	3	1
$g_{\mu\nu}$	$\longrightarrow$	h_{ij}	N^i	N

Arnowitt-Deser-Misner Action

$$S = \int \mathrm{d}t\, \mathrm{d}^3x\, N\sqrt{h}\left(K^{ij}K_{ij} - K^2 + R^{(3)} - 2\Lambda\right)$$

Extrinsic Curvature

$$K_{ij} \equiv \frac{1}{2}N^{-1}\left(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$$

$\longrightarrow$ Only the spatial metric h_{ij} is dynamical

Degrees of Freedom in GR

- 6 dynamical variables h_{ij} constrained by
- 4 spacetime gauge symmetries

6 - 4 = 2 degrees of freedom

- These are the polarizations of the graviton
- To avoid new degrees of freedom, balance dynamical variables against gauge symmetries

Approach

- GR is a theory of a spatial metric h_{ij} (which has six components) subject to four spacetime gauge symmetries

$$6 - 4 = 2$$

- We consider theories of a unit-determinant spatial metric $\tilde{h}_{ij}$ (which has five components) subject to three spatial gauge symmetries

$$5 - 3 = 2$$

- In a particular gauge, General Relativity can be cast in this form

What freedom is there to modify spatially covariant General Relativity?

Ultralocal Modified Gravity

- Neglect terms in action of quadratic or higher order in spatial derivatives
- This is a long-wavelength, deep infrared limit in which gravitons have only kinetic energy
- In this limit, consistency implies only one restriction: spatial conformal symmetry

$$x^i \longrightarrow x^i/\lambda$$

Local Modified Gravity

- Allow the action to depend on spatial derivatives through the Ricci scalar $\tilde{R}$ of $\tilde{h}_{ij}$
- Ricci scalar dependence is the leading local correction to infrared dynamics
- This class of theories includes GR
- In addition to spatial conformal symmetry, consistency requires the vanishing of a non-trivial field dependent vector quantity

$$\mathcal{A}_k = \frac{1}{2}\omega^2 \frac{\partial \pi_\omega}{\partial \tilde{R}} \left(2\tilde{\pi}^{ij}\tilde{\nabla}_i - \omega(\tilde{\nabla}^j \pi_\omega)\right) \left(\sum_{n=2}^{\infty} n\Pi(n-2)_{jk} \frac{\partial \pi_\omega}{\partial \phi(n)}\right)$$

$$- \frac{1}{3}\omega^2 \frac{\partial \pi_\omega}{\partial \tilde{R}} \tilde{\nabla}_k \left(\sum_{n=3}^{\infty} n\phi(n-1) \frac{\partial \pi_\omega}{\partial \phi(n)}\right) - \omega \left(\omega \frac{\partial \pi_\omega}{\partial \tilde{\pi}^{jk}} \tilde{\nabla}^j + \frac{4}{3}\tilde{\nabla}_k\right) \frac{\partial \pi_\omega}{\partial \tilde{R}}$$

Summary of Results

- We can modify GR without new DOF, provided the theory is invariant under spatial conformal symmetry
- In the ultralocal limit, the action is otherwise arbitrary
- Locality constrains the action through the consistency condition

$$\mathcal{A}_k = 0$$

Win-Win Situation

- Is GR the unique low-energy local, realistic theory of the graviton degrees of freedom?
- If so, Lorentz invariance in the gravitational sector could arise as an accidental symmetry
- If not, infrared modifications of the graviton could shed light on dark energy
- Either way, the results will be interesting!

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Thank you