Massive Gravity and Quasi-Dilaton: Theory and Cosmology

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Based on works with: C. de Rham, A. Tolley;
and G. D'Amico, S. Dubovsky, L. Hui, D. Pirtskhalava

August 24, 2012

Motivation:

While testing General Relativity (GR), good to have an alternative theory to compare with, and test both against the data. Brans-Dicke theory was introduced for that purpose in 1960s

Cosmic acceleration: a new physical scale of dark energy, 10^{-33} eV. This may be a scale at which gravity should be extended

Extension of GR by a mass term is arguably the best motivated modification. Yet, such an extension – being a fundamental question of field theory – had been a problem up until recently

GR extended with the mass and potential terms, evades S. Weinberg's no-go theorem for the old cosmological constant problem

GR Extended by Mass and Potential Terms

Previous no-go statements invalid: *de Rham, GG*

The Lagrangian of the theory: *de Rham, GG, Tolley*

Using $g_{\mu\nu}(x)$ and 4 scalars $\phi^a(x)$, $a = 0, 1, 2, 3$, define

$$\mathcal{K}^\mu_\nu(g, \phi) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}}$$

The Lagrangian is written using notation $tr(\mathcal{K}) \equiv [\mathcal{K}]$:

$$\mathcal{L}_{dRGT} = M^2_{\rm pl}\sqrt{g}\left(R + m^2\left(\mathcal{U}_2 + \alpha_3\,\mathcal{U}_3 + \alpha_4\,\mathcal{U}_4\right)\right)$$

$$\mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2]$$

$$\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]$$

$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]$$

Lagrangian Rewritten via Levi-Civita Symbols:

de Rham, GG, Heisenberg, Pirtskhalava (decoupling limit)
Th. Nieuwenhuizen (in the full theory)

$$\mathcal{L}_{dRGT} = M_{\rm pl}^2 \sqrt{g} \left(R + m^2 \left(\mathcal{U}_2 + \alpha_3 \, \mathcal{U}_3 + \alpha_4 \, \mathcal{U}_4 \right) \right)$$

$$\mathcal{U}_2 = \epsilon_{\mu\nu\alpha\beta} \epsilon^{\rho\sigma\alpha\beta} \mathcal{K}^\mu_\rho \mathcal{K}^\nu_\sigma$$

$$\mathcal{U}_3 = \epsilon_{\mu\nu\alpha\gamma} \epsilon^{\rho\sigma\beta\gamma} \mathcal{K}^\mu_\rho \mathcal{K}^\nu_\sigma \mathcal{K}^\alpha_\beta$$

$$\mathcal{U}_4 = \epsilon_{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\gamma\delta} \mathcal{K}^\mu_\alpha \mathcal{K}^\nu_\beta \mathcal{K}^\rho_\gamma \mathcal{K}^\sigma_\delta$$

$$\mathcal{K}^\mu_\nu(g, \phi) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha} \partial_\alpha \phi^a \partial_\nu \phi^b \eta_{ab}} \quad \text{unitary gauge} \quad \phi^a = x^a$$

Hamiltonian construction: *Hassan, Rachel A. Rosen*
Another proof: *Mirbabayi*

No flat FRW solution:

D'Amico, de Rham, Dubovsky, GG, Pirtskhalava, Tolley

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2, \quad \phi^0(t) = f(t), \quad \phi^j(x) = x^j$$

Minisuperspace Lagrangian (for $\alpha_{3,4} = 0$):

$$L = 3M_{\rm pl}^2 \left(-a\dot{a}^2 + m^2(2a^3 - 3a^2 + a) - m^2|\dot{f}|(a^3 - a^2) \right)$$

$$\frac{d}{dt}(m^2(a^3 - a^2)) = 0$$

No cosmology if m is a constant.

Exception: Open FRW selfaccelerated universe:

Gumrukcuoglu, Lin, Mykohyama

Possible ways to proceed for the flat universe:

(1) Heterogeneous and/or anisotropic cosmologies

(2) Field dependent mass $m \to m(\sigma)$: FRW solutions ok

Heterogeneous Solutions: Qualitative Picture

The Vainshtein radius for a domain of density ρ and size R

$$r_* = \left(\frac{\rho}{\rho_{co}}\right)^{1/3} R, \qquad \rho_{co} \equiv 3M_{\rm pl}^2 m^2$$

Within a patch of radius $1/m$, consider a typical Hubble volume, i.e., the volume enclosed by the sphere of radius

$$H^{-1} = \sqrt{\frac{3M_{\rm pl}^2}{\rho}}$$

This volume is in the Vainshtein regime, i.e., $r_* >> H^{-1}$, as long as

$$\rho >> \rho_{co}$$

Hence, should recovers FRW with great accuracy for $\rho >> \rho_{co}$!

Heterogeneous solutions: Quantitative Picture

$$ds^2 = -dt^2 + C(t,r)dtdr + A(t,r)^2(dr^2 + r^2 d\Omega^2),$$
$$\phi^0 = f(t,r), \quad \phi^j(x) = g(t,r)\frac{x^j}{r}$$

Einstein's equation extended with the mass and potential terms:

$$G_{\mu\nu} = m^2 X_{\mu\nu} + 8\pi G_N T_{\mu\nu}$$

Early universe: in the first approximation neglect $m^2 X_{\mu\nu}$, get FRW.
In the obtained FRW background solve for ϕ^a's

$$m^2 \nabla^{\mu}_{g_{FRW}} X_{\mu\nu}(g_{FRW}, \phi^a) = 0$$

Find ϕ^a's, and calculate backreaction to make sure that $m^2 X_{\mu\nu} << 8\pi G_N T_{\mu\nu}$. This is the case for $\rho >> \rho_{co}$.
What about the case when $\rho \sim \rho_{co}$?

Selfacceleration and pseudo-homogeneous solutions

In the dec limit: *de Rham, GG, Heisenberg, Pirtskhalava*
Exact solution: *Koyama, Niz, Tasinato (1,2,3)*
Other solutions: *M. Volkov; L. Berezhiani, et al; ...*

For instance, *Koyama-Niz-Tasinato* solution:

$$ds^2 = -d\tau^2 + e^{m\tau}(d\rho^2 + \rho^2 d\Omega^2)$$

while, ϕ^0 and ϕ^ρ, are inhomogeneous functions

$$\mathrm{arctanh}\left(\frac{\sinh(m\tau/2) + e^{m\tau/2}m^2\rho^2/8}{\cosh(m\tau/2) - e^{m\tau/2}m^2\rho^2/8}\right), \quad \rho e^{m\tau/2}$$

Selfacceleration with heterogeneous metric: *Gratia, Hu, Wyman*
Selfacceleration seems to be a generic feature of this theory
However, vanishing of kinetic terms for extra modes seems to be a generic feature of these solutions, *D'Amico* (see, more discussions on this subtle issue later).

Theory of Quasi-Dilaton: *D'Amico, GG, Hui, Pirtskhalava*

Invariance of the action to rescaling of the reference frame coordinates ϕ^a w.r.t. the physical space coordinates, x^a, requires a field σ. In the Einstein frame:

$$\phi^a \to e^{\alpha}\ \phi^a, \qquad \sigma \to \sigma - \alpha M_{\rm Pl}$$

Hence we can construct the invariant action by replacing $\mathcal{K}$ by $\bar{K}$

$$\bar{K}^{\mu}_{\nu} = \delta^{\mu}_{\nu} - e^{\sigma/M_{\rm Pl}}\sqrt{g^{\mu\alpha}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}}$$

and adding the sigma kinetic term

$$\mathcal{L} = \mathcal{L}_{dRGT}\left(\mathcal{K} \to \bar{K}\right) - \omega\sqrt{g}(\partial\sigma)^2$$

In the Einstein frame σ does not couple to matter, but it does in the Jordan frame

Cosmology of Quasi-Dilaton: Flat FRW Solutions

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 \quad \phi^0 = f(t), \quad \phi^i = x^i, \quad \sigma = \sigma(t)$$

Friedmann equation:

$$\begin{aligned} 3M_{\rm Pl}^2 H^2 \quad &= \frac{\omega}{2}\dot{\sigma}^2 + \rho_m + \\ 3M_{\rm Pl}^2 m^2 \quad &\left[c_0 + c_1 \left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right) + c_2 \left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right)^2 + c_3 \left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right)^3 \right] \end{aligned}$$

Constraint equation:

$$q_0 + q_1 \left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right) + q_2 \left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right)^2 + q_3 \left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right)^3 = \frac{k e^{-\sigma/M_{\rm Pl}}}{a^3} .$$

Particular Solutions for $k = 0$:

$$\left(\frac{e^{\sigma/M_{\rm Pl}}}{a} \right) = c, \quad \dot{\sigma} = M_{\rm Pl} H$$

Friedmann equation

$$(3 - \frac{\omega}{2}) M_{\rm Pl}^2 H^2 = \rho_m + 3 M_{\rm Pl}^2 m^2 \left[c_0 + c_1 c + c_2 c^2 + c_3 c^3 \right]$$

Constraint equation

$$q_0 + q_1 c + q_2 c^2 + q_3 c^3 = 0$$

Determine $f(t)$ from the sigma equation:

$$a\dot{f} = 1 + \frac{\omega}{3\kappa m^3} (3H^2 + \dot{H})$$

Small Perturbations:

Unitary gauge, ϕ^a's are frozen to their background values

$$g_{\mu\nu} = a^2(\eta_{\mu\nu} + h_{\mu\nu}(t,x)), \quad \sigma = ln(ca) + \zeta(t,x)$$

No diff invariance for $h_{\mu\nu}$ as long as ϕ^a's are frozen

Lagrangian density in conformal coordinates

$$a^4 \quad \left(\frac{\omega}{a^2}((\zeta')^2 - (\partial_j\zeta)^2) + \frac{\omega H}{a}(h_{00} + h_{jj})\zeta' - \frac{2\omega H}{a} h_{0j}\partial_j\zeta\right) \quad +$$
$$a^4 \quad ((\gamma_1 h_{00} + \gamma_2 h_{jj})\zeta + \gamma_3 h_{00}^2 + \gamma_4 h_{00} h_{jj} + \gamma_5 h_{0j} h_{0j} + \gamma_6 h_{ij} h_{ij} + \gamma_7 h_{jj}^2)$$

Need to check that there is no BD ghost – should be absent by construction, selfconsistency check. Need to check that all the other modes are good

Perturbations in the Decoupling Limit:

$$h_{\mu\nu} = \bar{h}_{\mu\nu} + \frac{1}{m}(\nabla_\mu A_\nu + \nabla_\nu A_\mu), \quad A_j = S_j^T + \partial_j b$$

$$m \to 0, \quad H \to 0, \quad H/m = \mathit{fixed\ number}$$

Quadratic part of the Lagrangian density for scalars

$$\omega(\dot{\zeta}^2 - (\partial_i \zeta)^2) + p(\partial_j A_0)^2 + 2qA_0 \Delta \dot{b} + r(\partial_0 \partial_j b)^2 + 4\gamma_{6,7}(\Delta b)^2$$

A_0 is nondynamical (would be BD ghost, which is absent here)
Integrating out A_0, and collecting coefficients in front of time derivatives of b one gets

$$(r - \frac{q^2}{p})(\partial_0 \partial_j b)^2 = 0$$

The only dynamical field in the scalar sector is quasi-dilaton, ζ.

Possible Cures for Vanishing Kinetic Terms?

- Is there a symmetry to remove these degrees of freedom? Nonlinear symmetry?
- Some degrees of freedom may cease to be dynamical on certain backgrounds (including at nonlinear level). Could this be the case for some of the selfaccelerated solutions?
- In the context of quantum effective field theory: vanishing of the kinetic terms at the scale Λ_3; RG running below this scale will induce the kinetic terms with logarithmic coefficients if there is no symmetry; can this work?
- If non of the above works, then truly inhomogeneous and non-isotropic backgrounds?

Conclusions:

- A classical theory that extends GR by the mass and potential term is available now; many questions of astrophysics and cosmology can be studies and comparisons can be made with GR as well as with data
- Generic cosmological solutions have no FRW symmetries, but can approximate well FRW cosmologies in the early universe
- Selfaccelerated solutions seem to be a generic feature; but fluctuations exhibit behavior that needs to be understood to see if these solutions are acceptable
- Further extension of the theory are possible, I discussed quasi-dilaton, and showed that solutions with FRW symmetries come back, while selfacceleration is also present. General tensorial extensions by *Hinterbichler and R.A. Rosen*
- Quantum aspects not yet well understood (work to follow, hopefully soon) however, don't see a reason to postpone investigation of interesting classical issues