Matter Interactions in Effective Field Theories of Dark Energy

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Variety is the Spice of Life

- Quintessence
- k-essence
- Brans-Dicke theories
- Ghost Condensate
- Extra-dimensions a la UED/RS/ADD/DGP
- $f(R)$ gravity
- Gauss-Bonnet gravity
- ...

- In a low energy four-dimensional limit, all these theories essentially behave as GR + scalar field(s)
- Perhaps we can perform a general analysis in this regime?

Systematic Characterization

- **Effective Field Theory of gravity + scalar field** (eg, Weinberg, JB and Eanna Flanagan)
  - Generalizing inflationary models to allow for matter couplings
  - Identifying regimes of validity and constraints on UV theory
- Effective Field Theory of perturbations to FRW (eg, Creminelli *et al*.)
  - Incorporating interactions with matter in perturbative descriptions

Our Approach

Leading Order Action: GR + Canonical Scalar (Quintessence) Field

$$S_0 = \int d^4x\sqrt{-g}\left\{\frac{m_p^2}{2}R - \frac{1}{2}(\nabla\phi)^2 - U(\phi)\right\} + S_{\rm matter}\left[e^{\alpha(\phi)}g_{\mu\nu}, \{\psi\}\right]$$

Perturb the Action

$$\phi, g^{\mu\nu}, R_{\mu\nu\sigma\lambda}, \epsilon_{\mu\nu\sigma\lambda}, T_{\mu\nu}, \nabla_\mu, \Box \ldots$$

Rules of Analysis

- Use a derivative expansion to fourth order
- Remove higher order derivatives in equations of motion ("reduce" the action)
- Impose the Weak Equivalence Principle (Note: not a symmetry of the theory)

EFT Considerations

- Can use a pseudo-Nambu-Goldstone Boson (pNGB) construction to ensure light quintessence field
- pNGB construction yields expansion rules
- Expansion parameter given by

$$\frac{H_0^2}{M^2} \ll 1$$

  for some mass scale M of fields integrated out
- Also specifies scaling of operators. For an operator with d derivatives, mass dimension n, scaling is

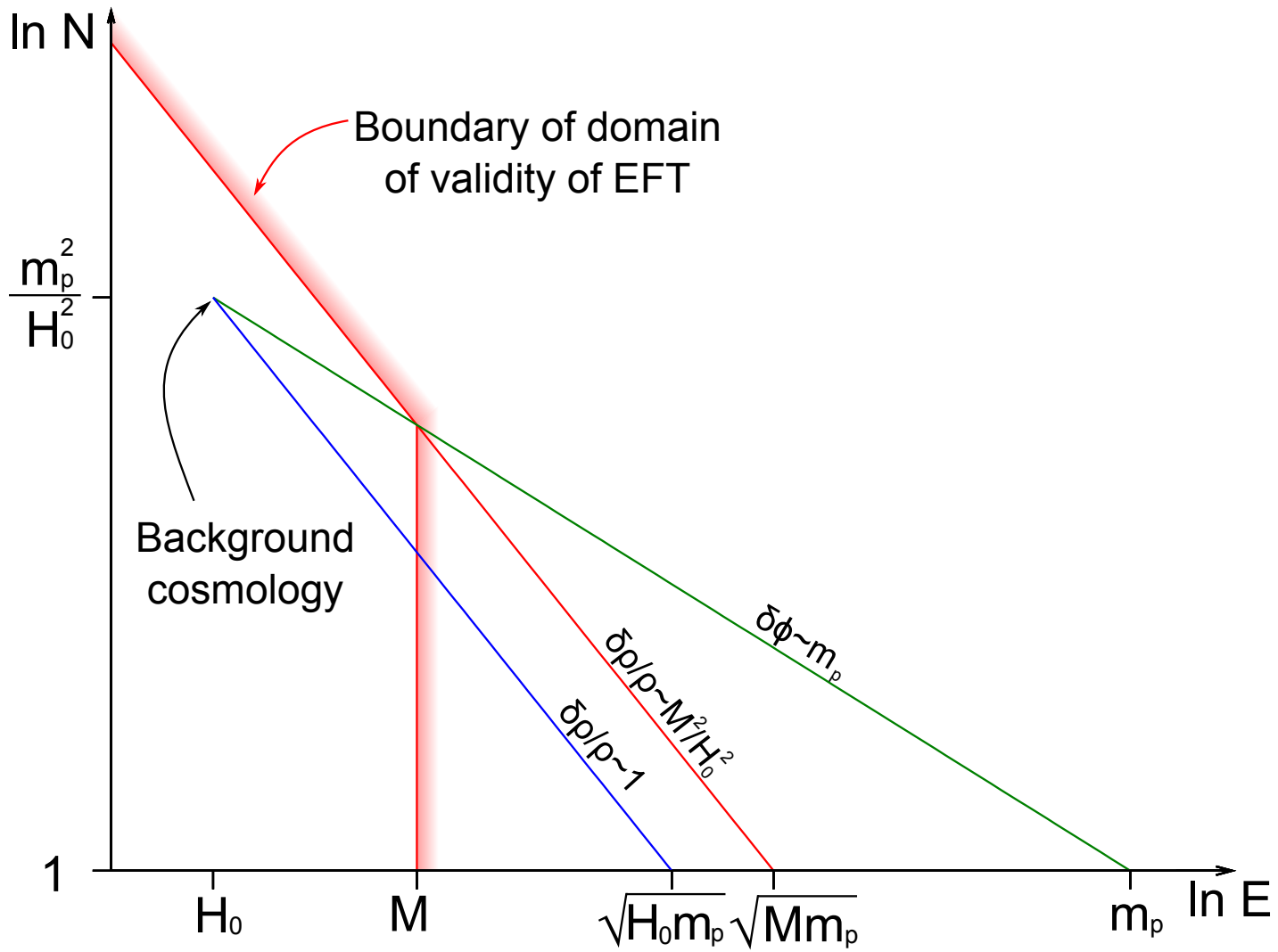
$$\sim M^{2-d} m_p^{2+d-n}$$

Results

$$S = \int d^4x\sqrt{-g}\left\{\frac{m_p^2}{2}R - \frac{1}{2}(\nabla\phi)^2 - U(\phi) + a_1(\nabla\phi)^4\right.$$
$$+ b_2 T(\nabla\phi)^2 + c_1 G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$$
$$+ d_3\left(R^2 - 4R^{\mu\nu}R_{\mu\nu} + R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}\right)$$
$$+ d_4\epsilon^{\mu\nu\lambda\rho}C_{\mu\nu}{}^{\alpha\beta}C_{\lambda\rho\alpha\beta}$$
$$\left.+ e_1 T^{\mu\nu}T_{\mu\nu} + e_2 T^2\right\}$$
$$+ S_m\left[e^{\alpha(\phi)}g_{\mu\nu}\right]$$

- Coefficients are functions of ϕ with specific scalings
- Parameter space is given by nine free functions

Regime of Validity



Utility of this Approach

- Can describe background and perturbative evolution of the cosmology
- Radiative corrections under control, given constraints on UV theory
- Within regime of validity, yields a very general description

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Perturbative Analysis

- An EFT of perturbations about FRW has proved useful for inflation (Cheung *et al*.) as well as quintessence (Creminelli *et al*.)
- Background evolution must be specified
- Perturbative description more powerful
- Existing framework needs extending to treat dark energy-matter interactions generally

Idea of EFT of Inflation

- Assume a single (effective) scalar field is responsible for dark energy, with FRW background evolution
- Choose a foliation of spacetime based on $\phi(t) = \text{const}$
- Time diffeomorphism symmetry is broken (metric eats the scalar field)
- Identify operators which respect remaining symmetry
- Separate operators into leading order and perturbative operators
- Construct EFT action

Idea of EFT of Inflation

$$S = \int d^4x\sqrt{-g}\Bigg\{\frac{m_p^2}{2}R + \Lambda(t) + c(t)g^{00} + F^{(2)}(\delta g^{00}, \delta K_{\mu\nu}, \delta R_{\mu\nu\sigma\lambda}; t, \delta^0_\mu, g^{\mu\nu}, \epsilon^{\alpha\beta\gamma\delta})\Bigg\}$$

Idea of EFT of Inflation

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- Fix background evolution $a(t)$ by specifying $c(t)$, $\Lambda(t)$
- Restore diffeomorphism symmetry using Stückelberg trick

Stückelberg Trick

- Consider the situation of a massive photon field:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu$$

- The gauge invariance $A_\mu \to A_\mu + \partial_\mu \lambda$ is broken by the mass term.

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- The gauge invariance $A_\mu \to A_\mu + \partial_\mu \lambda$ is broken by the mass term.
- However, it can be restored by introducing in the action a field π which transforms as $\pi \to \pi - \lambda$ under the gauge transformation.

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} (A_\mu + \partial_\mu \pi)(A^\mu + \partial^\mu \pi)$$

Stückelberg Trick

- Longitudinal (scalar) component of the photon field is captured by the scalar field, and the action is again gauge invariant.
- Same trick can be used to restore diffeomorphism invariance.

Matter Couplings - Conformal Coupling

- Metric which the matter couples to can be conformally scaled

$$S_m\left[e^{\alpha(\phi)}g_{\mu\nu},\{\psi\}\right]$$

Extend EFT of Inflation by working in Jordan frame

$$S=\int d^4x\sqrt{-g}\Bigg\{f(t)\frac{m_p^2}{2}R+\Lambda(t)+c(t)g^{00}+F^{(2)}(\delta g^{00},\delta K_{\mu\nu},\delta R_{\mu\nu\sigma\lambda};t)\Bigg\}+S_m\left[g_{\mu\nu},\{\psi\}\right]$$

Matter Couplings - Conformal Coupling

- In Einstein frame, we can exploit the stress-energy tensor

$$S_m[g_{\mu\nu}, \{\psi\}] = S_0 - \frac{1}{2}\int d^4x\sqrt{-g}T_{\mu\nu}\delta g^{\mu\nu}$$

- Interested in quintessence perturbations coupling to matter perturbations
- When conformal factor is introduced,

$$S_m\left[e^{\alpha(\phi)}g_{\mu\nu}\right] = S_0 - \frac{1}{2}\int d^4x\sqrt{-g}e^{2\alpha(\phi)}\left[(T_{\mu\nu} - T^0_{\mu\nu})\delta\left(e^{-\alpha(\phi)}g^{\mu\nu}\right) + T^0_{\mu\nu}\delta\left(e^{-\alpha(\phi)}g^{\mu\nu}\right)\right]$$

- $\alpha(\phi) = \alpha(t) \rightarrow \alpha(t+\pi)$

Matter Couplings - Stress Energy Tensor

- Stress-Energy Tensor terms need some representation in EFT of Inflation

Extra terms describe any stress-energy tensor dependency

$$S = \int d^4x\sqrt{-g}\Big\{ f(t)\frac{m_p^2}{2}R + \Lambda(t) + c(t)g^{00} + g(t)T^{00} + h(t)T + F^{(2)}(\delta g^{00}, \delta K_{\mu\nu}, \delta R_{\mu\nu\sigma\lambda}, \delta T_{\mu\nu}; t)\Big\} + S_m[g_{\mu\nu}, \{\psi\}]$$

Summary

- Have constructed an effective field theory to describe dark energy
- Framework for investigating perturbative behavior is in progress
- Hope to constrain parameters in general descriptions, based on cosmological history and the behavior of cosmic perturbations