Structure formation in modified gravity models

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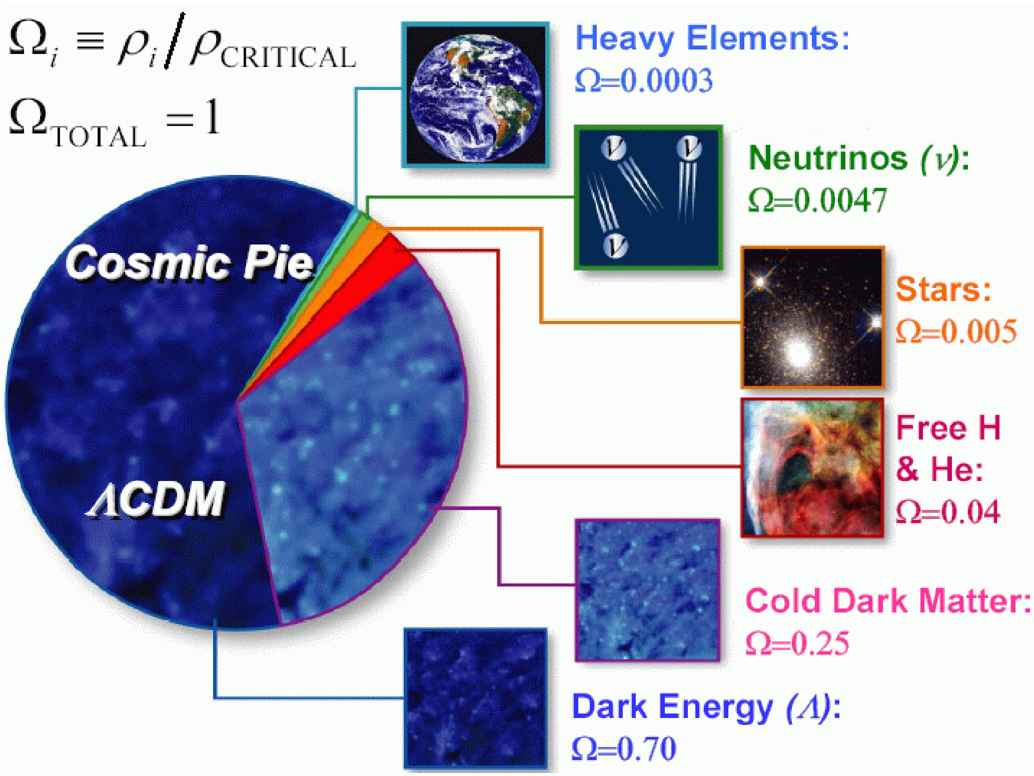
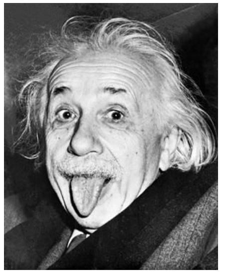
Dark energy v modified gravity



Is cosmology probing the breakdown of general relativity at large distance?

General relativity

- Why do we believe general relativity?



  - Observational point of view

    GR is tested to very high accuracy by solar system experiments and pulsar timing measurements

    C.Will gr-qc/0510072

  - Theoretical point of view

    GR is the unique metric theory in 4D that gives second order differential equations

FIFTH FORCE

FIFTH FORCE ENERGY IS THE BUILDING BLOCK ON WHICH ALL LIFE IS CREATED AND IS THE ESSENCE OF LIFE ITSELF. FIFTH FORCE IS IN FACT MATTER BUT IT CANNOT BE SEEN AND THIS MATTER IS THE NATURE OF THE PRIME. OUT OF ALL THE ENERGIES IN THE UNIVERSE FIFTH FORCE IS THE MOST POWERFUL ENERGY OF ALL BECAUSE IT IS IN EVERY LIFE THING AND IT IS IN EVERY LIFE BEING WHETHER THEY ARE OF THE HUMAN RACE OR AN ALIEN RACE LIVING ON A PLANET IN A FAR AND DISTANT GALAXY IN OUR UNIVERSE OR IN A PARALLEL UNIVERSE LIGHT YEARS WAY.

- NEW WEBSITE COMING SOON...

Brans-Dicke theory

- Action

$$S = \int d^4x \left(\psi R - \frac{\omega_{BD}}{\psi} (\nabla \psi)^2 + V(\psi) \right) \qquad V \sim H_0^2 M_{pl}^2$$

f(R) gravity: $\omega_{BD} = 0$

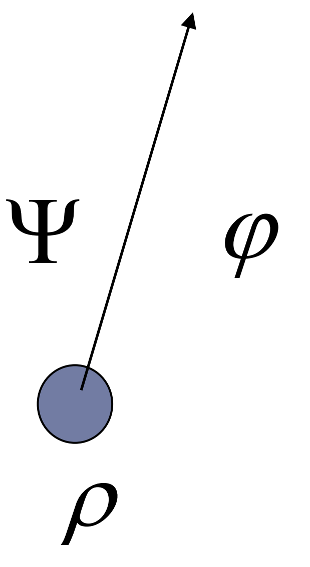
quasi-static approximations (neglecting time derivatives)

$$ds^2 = -(1+2\Psi)dt^2 + a(t)^2(1-2\Phi)d\vec{x}^2 \quad \psi = \psi_0 + \varphi$$

$$(3+2\omega_{BD})\nabla^2\varphi = -8\pi G\rho$$

$$\nabla^2\Psi = 4\pi G\rho - \frac{1}{2}\nabla^2\varphi$$

$$\Phi - \Psi = -\varphi$$



Constraints on BD parameter

- Solutions

$$(3+2\omega_{BD})\nabla^2\varphi = -8\pi G\rho$$

$$\nabla^2\Psi = -4\pi G\left(\frac{4+2\omega_{BD}}{3+2\omega_{BD}}\right)\rho, \quad G_{eff} = \left(\frac{4+2\omega_{BD}}{3+2\omega_{BD}}\right)G$$

$$\Psi = \frac{2+\omega_{BD}}{1+\omega_{BD}}\Phi \equiv \gamma^{-1}\Phi$$

- PPN parameter

$$\gamma = \frac{1+\omega_{BD}}{2+\omega_{BD}}$$

$$\gamma - 1 = (2.1 \pm 2.3)\times 10^{-5} \qquad \omega_{BD} \geq 40,000$$

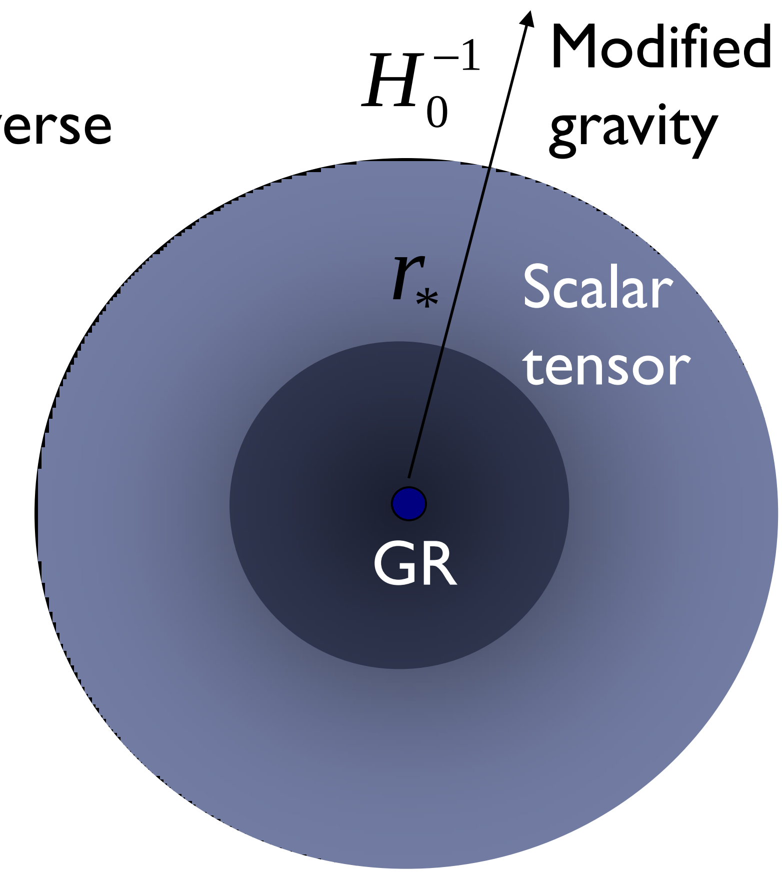
This constraint excludes any detectable modifications in cosmology

General picture

- Largest scales
  gravity is modified so that the universe accelerates without dark energy

- Large scale structure scales
  gravity is still modified by a fifth force from scalar graviton

- Small scales (solar system)
  GR is recovered by ***"screening mechanism"***



How to suppress the fifth force (1)

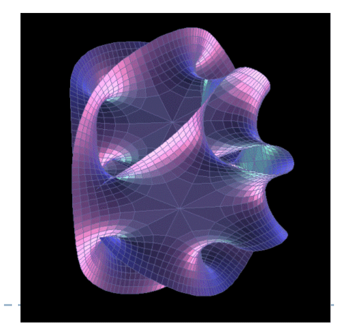
$$S=\int d^4x\left(\psi R-\frac{\omega_{BD}(\psi)}{\psi}(\nabla\psi)^2+V(\psi)+L_m[g]\right)$$

GR is recovered if

(i) the mass is large $V''\rightarrow\infty$

(ii) the kinetic term is large $\omega_{BD}\rightarrow\infty$

These limits should be realised in environmentally (density) dependent way to avoid the recovery of GR on all scales

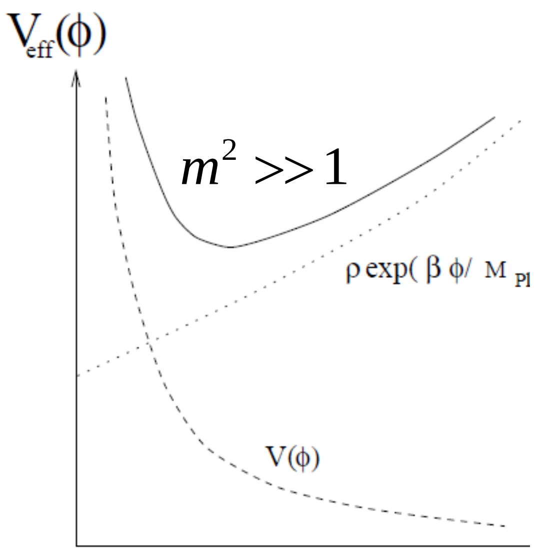


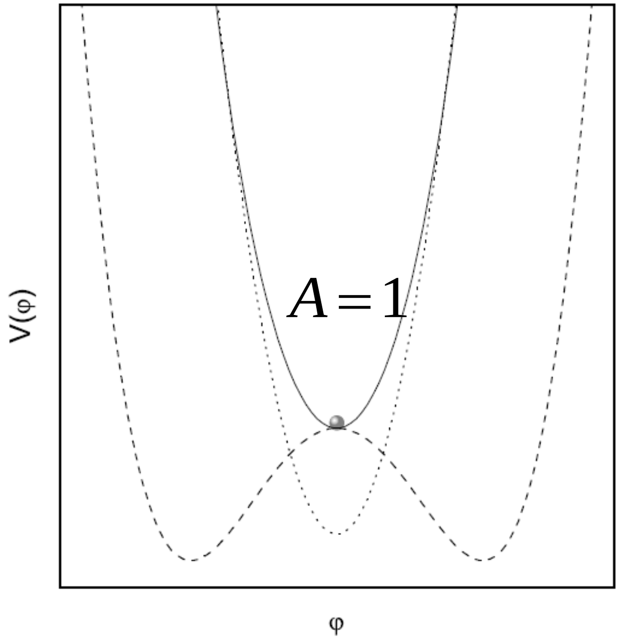
Chameleon/symmetron/dilaton

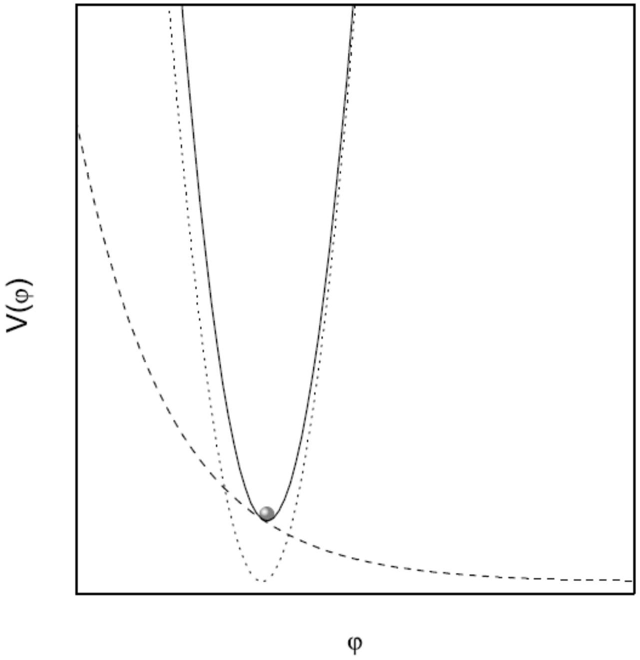
- Einstein frame

$$S_E = \int d^4x \left[\sqrt{-\bar{g}} \left(\bar{R} - \frac{1}{2} (\bar{\nabla}\phi)^2 + \bar{V}(\phi) \right) + L_m[A(\phi)^2 \bar{g}_{\mu\nu}] \right] \qquad A \leftrightarrow \omega_{BD}$$

$$\Box\phi = V_{eff}(\phi), \quad V_{eff}(\phi) = \bar{V}(\phi) - (A(\phi) - 1)\rho$$

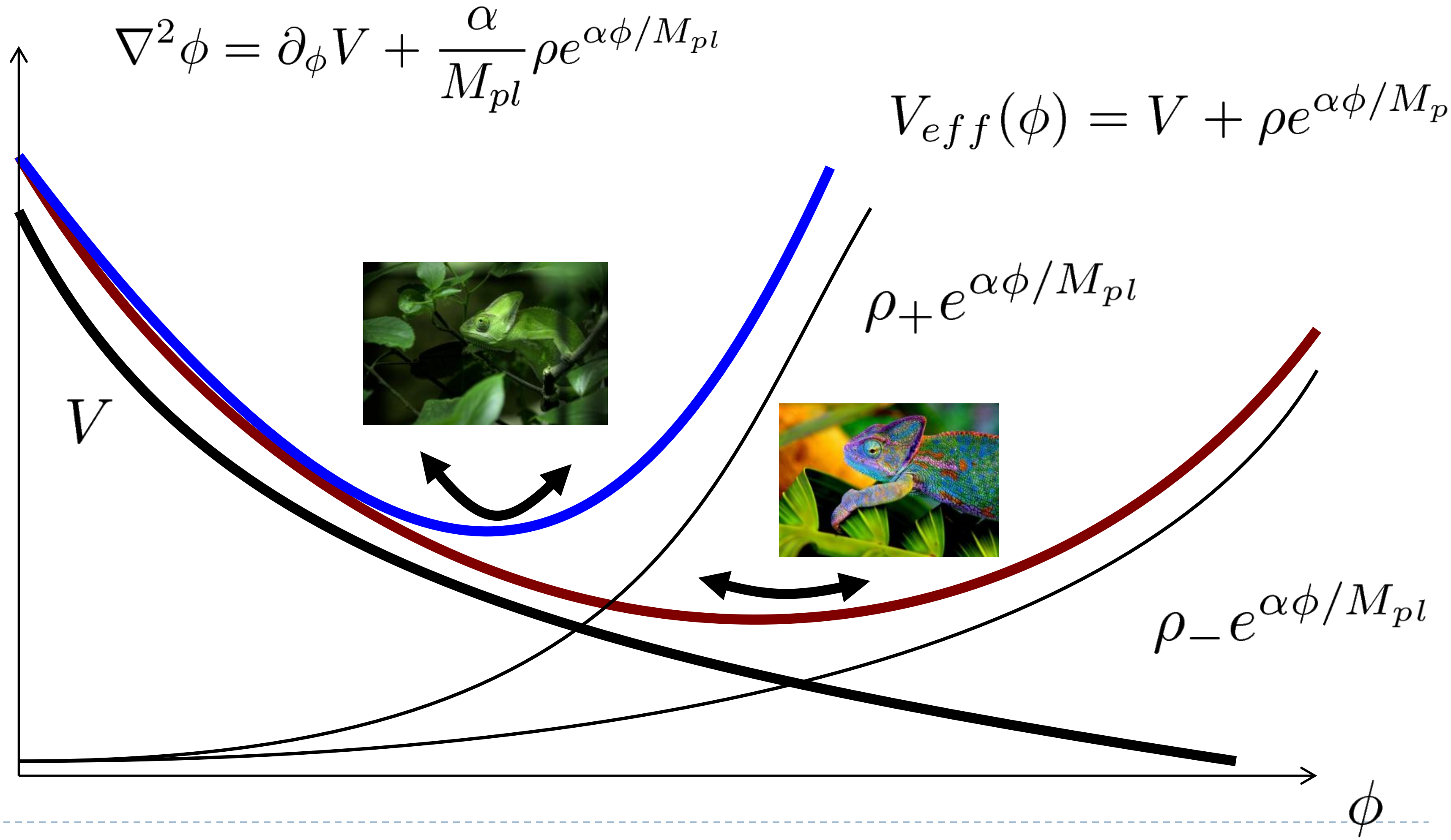






How we recover GR on small scales

- Chameleon mechanism (Khoury & Weltman)



How to suppress the fifth force (2)

- Vainshtein mechanism

  originally discussed in massive gravity

  rediscovered in DGP brane world model

  linear theory $\omega_{BD} = 0$

$$3\nabla^2\varphi = -8\pi G\rho$$

$$\nabla^2\Psi = 4\pi G\rho - \frac{1}{2}\nabla^2\varphi$$

  even if gravity is weak, the scalar can be non-linear

$$3\nabla^2\varphi + r_c^2\left\{\left(\nabla^2\varphi\right)^2 - \partial_i\partial_j\varphi\,\partial^i\partial^j\varphi\right\} = 8\pi G a^2\rho \qquad r_c \sim H_0^{-1}$$

Vainshtein mechanism

- Spherically symmetric solution for the scalar

$$\frac{d\varphi}{dr} = \frac{r_g}{r^2}\Delta(r), \quad \Delta(r) = \frac{2}{3}\left(\frac{r}{r_V}\right)^3\left(\sqrt{1+\left(\frac{r_V}{r}\right)^3} - 1\right) \quad r_V = \left(\frac{8r_c^2 r_g}{9}\right)^{\frac{1}{3}}, \quad r_g = 2GM$$

r_g $\qquad$ r_V $\qquad$ r_c

$$\Phi = \frac{r_g}{2r} + \sqrt{\frac{r_g r}{2r_c^2}}, \qquad \Phi = \frac{r_g}{2r}\left(\frac{2}{3}\right),$$

$$\Psi = -\frac{r_g}{2r} + \sqrt{\frac{r_g r}{2r_c^2}} \qquad \Psi = -\frac{r_g}{2r}\left(\frac{4}{3}\right)$$

2.95km $\qquad$ 0.1 kpc $\qquad$ 3000Mpc for the Sun

Solar system constraints

- The fractional change in the gravitational potential $\varepsilon = \dfrac{\delta\Psi}{\Psi}$

  The anomalous perihelion precession

$$\delta\phi = \pi r \frac{\partial}{\partial r}\left[r^2 \frac{\partial}{\partial r}\left(\frac{\varepsilon}{r}\right)\right]$$

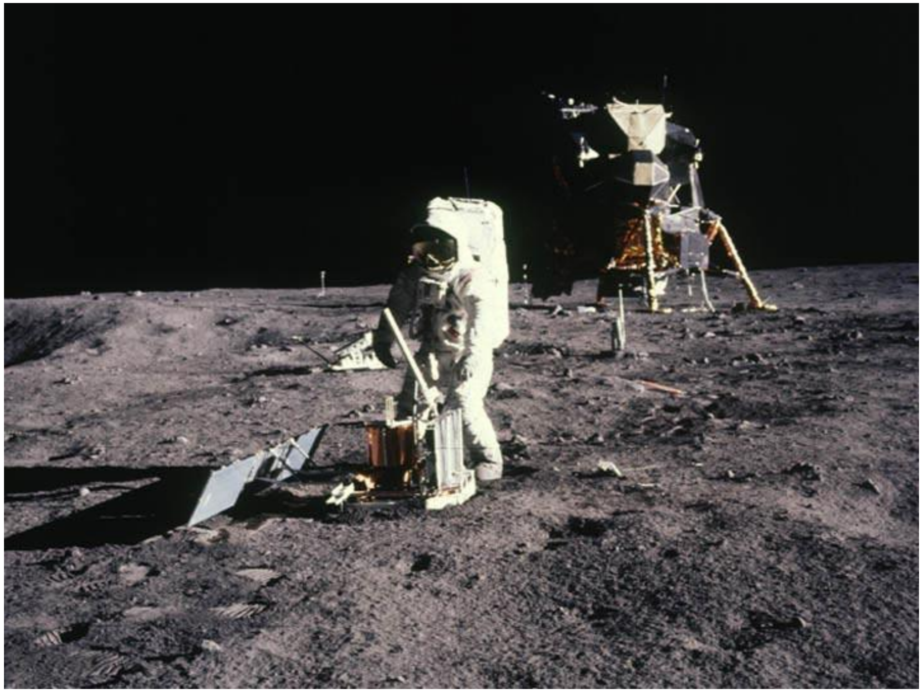
  The vainshtein radius is shorter for a smaller object

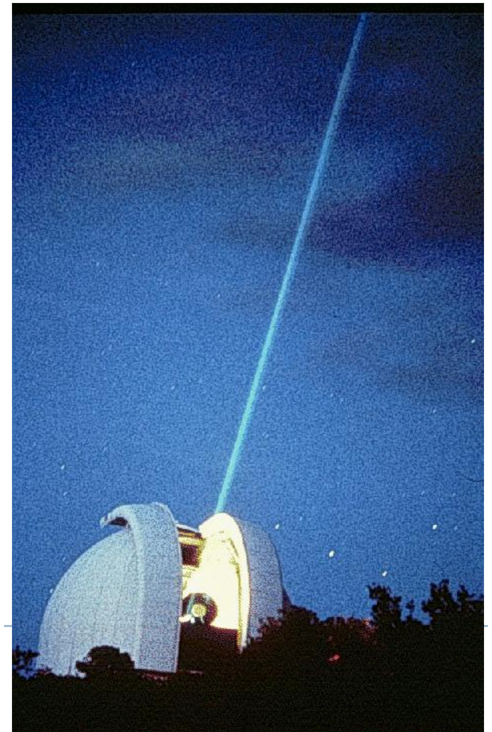
  Lunar laser ranging: the Erath-moon distance $r_{E-M} = 4.1\times 10^5\,\mathrm{km}$

$$\delta\phi = \frac{3\pi M_{pl}}{4}\left(\frac{r_{E-M}{}^3}{8\pi r_c^2 M_\oplus}\right)^{1/2} < 2.4\times 10^{-11}, \quad r_{E-M} << r_V$$

$$\Rightarrow \quad r_c > H_0^{-1}$$


Lunakhod 1
Lunakhod 2
Apollo XV
Apollo XIV
Apollo XI
SCIENCEphotoLIBRARY





Generalisaitons

- Galileon models $\partial_\mu \pi \to \partial_\mu \pi + c_\mu$ Nicolis, Rattazzi, Trincherini

$$\mathcal{L}_\pi = \sum_{i=1}^{5} c_i \, \mathcal{L}_i$$

$$\Pi^\mu{}_\nu \equiv \partial^\mu \partial_\nu \pi$$

$$[\Pi] \, \partial\pi \cdot \partial\pi \equiv \Box\pi \, \partial_\mu \pi \partial^\mu \pi$$

$$\begin{aligned}
\mathcal{L}_1 &= \pi \\
\mathcal{L}_2 &= -\tfrac{1}{2} \, \partial\pi \cdot \partial\pi \\
\mathcal{L}_3 &= -\tfrac{1}{2} \, [\Pi] \, \partial\pi \cdot \partial\pi \\
\mathcal{L}_4 &= -\tfrac{1}{4} \big([\Pi]^2 \, \partial\pi \cdot \partial\pi - 2 \, [\Pi] \, \partial\pi \cdot \Pi \cdot \partial\pi - [\Pi^2] \, \partial\pi \cdot \partial\pi + 2 \, \partial\pi \cdot \Pi^2 \cdot \partial\pi \big) \\
\mathcal{L}_5 &= -\tfrac{1}{5} \big([\Pi]^3 \, \partial\pi \cdot \partial\pi - 3[\Pi]^2 \, \partial\pi \cdot \Pi \cdot \partial\pi - 3[\Pi][\Pi^2] \, \partial\pi \cdot \partial\pi + 6[\Pi] \, \partial\pi \cdot \Pi^2 \cdot \partial\pi \\
&\quad +2[\Pi^3] \, \partial\pi \cdot \partial\pi + 3[\Pi^2] \, \partial\pi \cdot \Pi \cdot \partial\pi - 6 \, \partial\pi \cdot \Pi^3 \cdot \partial\pi \big)
\end{aligned}$$

- Massive gravity models

  Galileon models are naturally realised in the decoupling limit De Rham, Gabadadze, Tolley

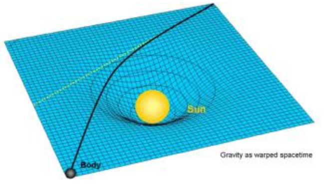
phenomenology

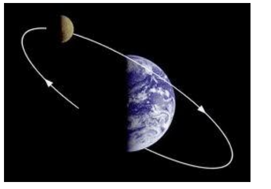
(1) Environment dependent screening

- In modified gravity models, dynamical mass inferred from velocity dispersions and lensing mass can be different

$$k^2(\Phi+\Psi)/2 = 4\pi G a^2 \Sigma(k,a)\rho_m\Delta_m$$

$$k^2\Psi = 4\pi G a^2 \mu(k,a)\rho_m\Delta_m$$





- f(R) $\Sigma \simeq 1$ The fifth force does not change geodesics of photon

  $\mu = [1:4/3]$ The fifth force enhances Newtonian gravity

- Difference between dynamical and lensing masses

$$\Delta_M(r) = \frac{d\Psi(r)/dr}{d\Psi_+(r)/dr} - 1, \quad \Psi_+ = (\Phi+\Psi)/2 \qquad \Delta_M = [0:1/3]$$

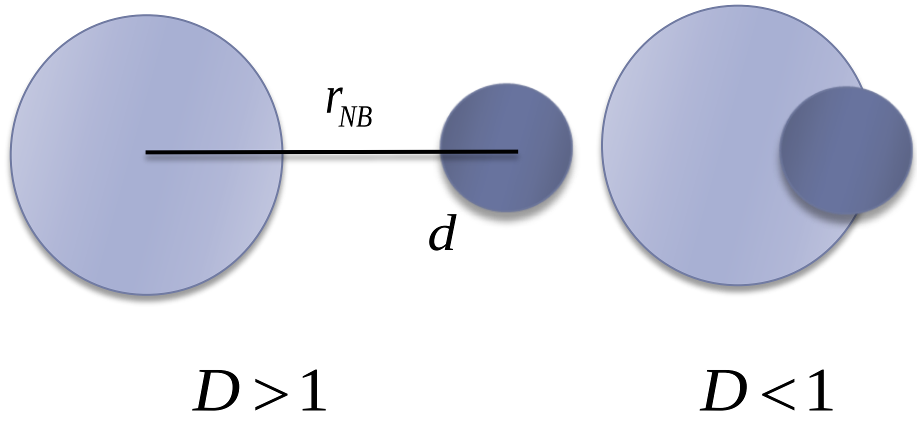
- Schmidt 1003.0409

Environmental effects

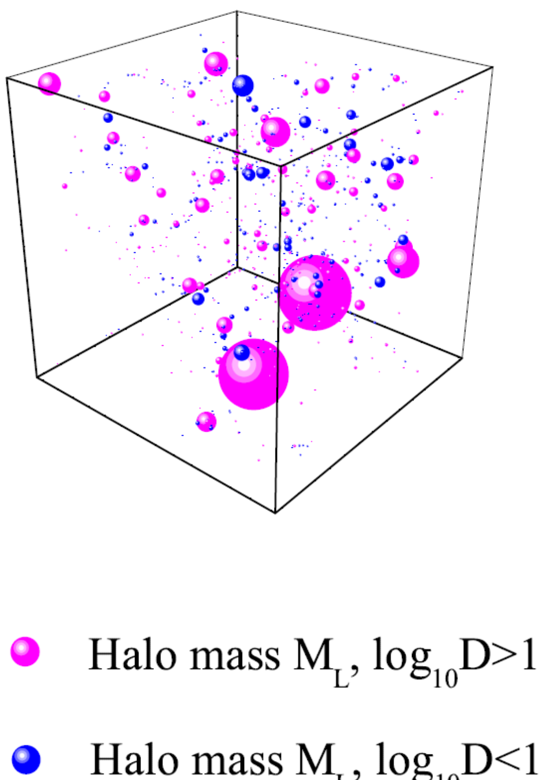
Zhao, Li, Koyama 1011.1257

Environment

$$D = d / r_{NB}, \quad M_{NB} > M$$



Halo mass

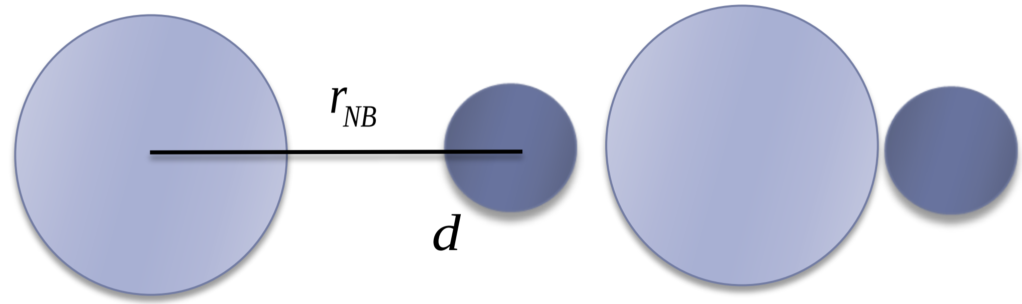


Environmental effects

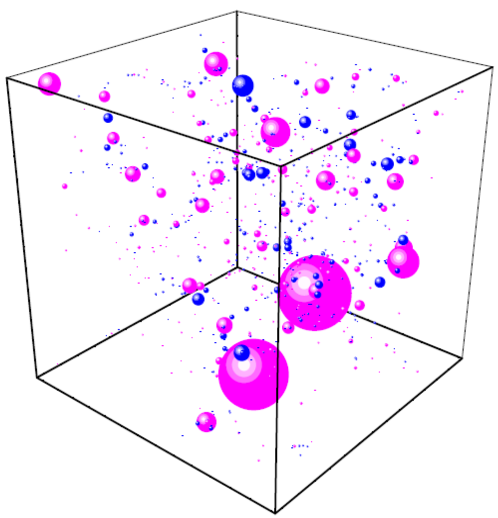
Zhao, Li, Koyama 1011.1257

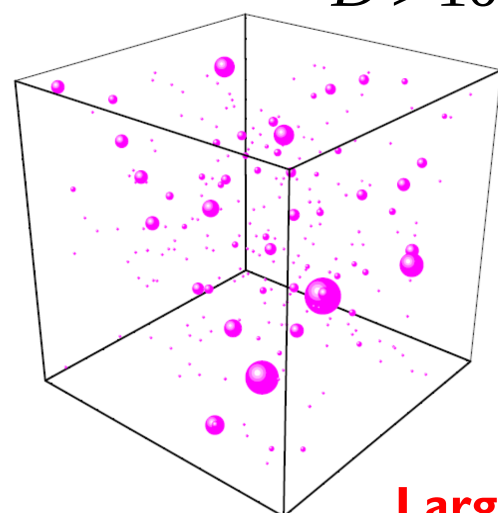
GR is recovered in large halos / dense environment

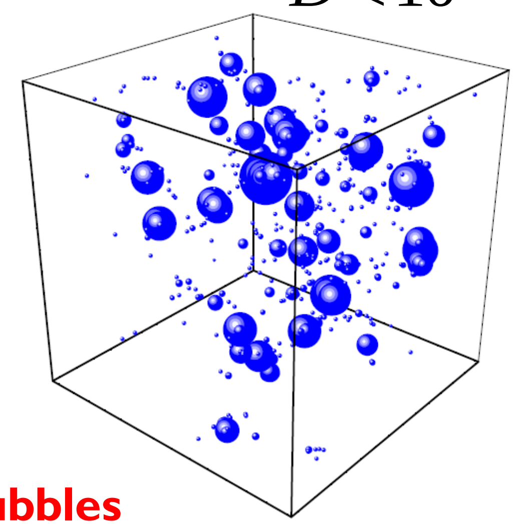
$$D = d / r_{NB}, \quad M_{NB} > M$$



$D > 10$ $\qquad$ $D < 10$







Large bubbles =better screened (GR is recovered)

Halo mass M_L, $\log_{10}D>1$

Halo mass M_L, $\log_{10}D<1$

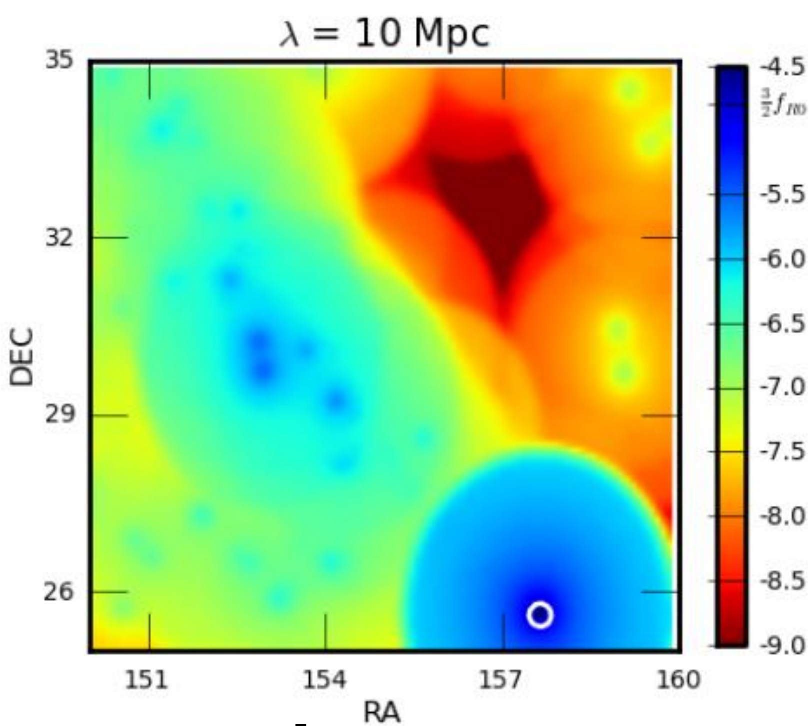
$|\log_{10}\Delta_M|$ isolated halos, $\log_{10}D>1$

$|\log_{10}\Delta_M|$ clustered halos, $\log_{10}D<1$

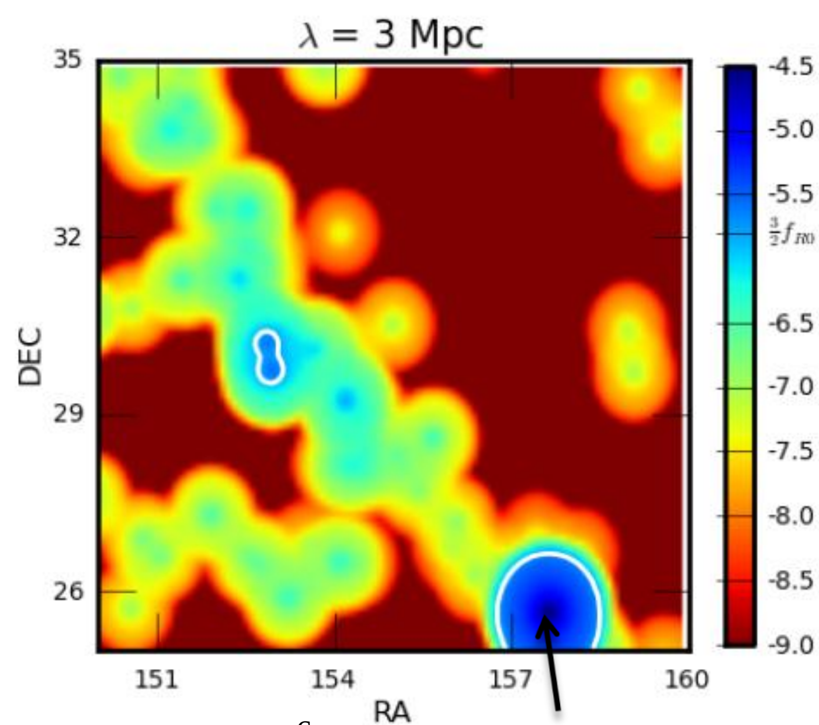
Creating a screening map

- It is essential to find places where GR is not recovered
  - Small galaxies in underdense regions
  - SDSS galaxies within 200 Mpc

Cabre, Vikram, Zhao, Jain, KK
1204.6046



$|f_{R0}| = 10^{-5}$



$|f_{R0}| = 10^{-6}$

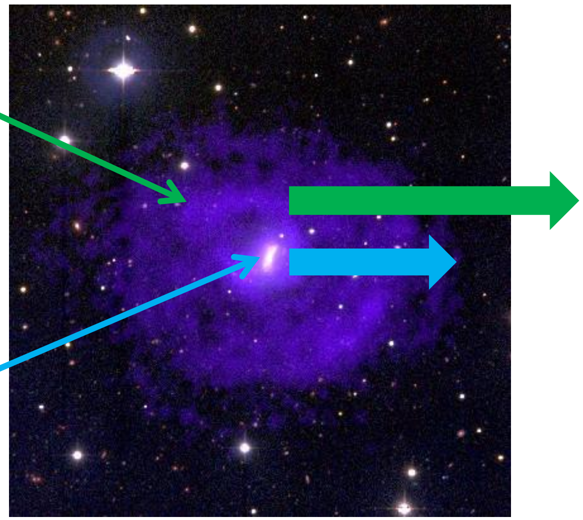
GR is recovered

Astrophysical tests

- Apparent violation of equivalent principle

HI gas:
unscreened

Stellar disk:
screened



Hui, Nicolis, Stubbs 0905.2966
Jain & VanderPlas 1106.0065

  - The rotation curve of HI gas is enhanced compare with stars
  - The stellar disk is displaced from the HI gas disk

This happens only in unscreened galaxies

(i.e. dwarf galaxies in voids)

(2) Vainshtein mechanism

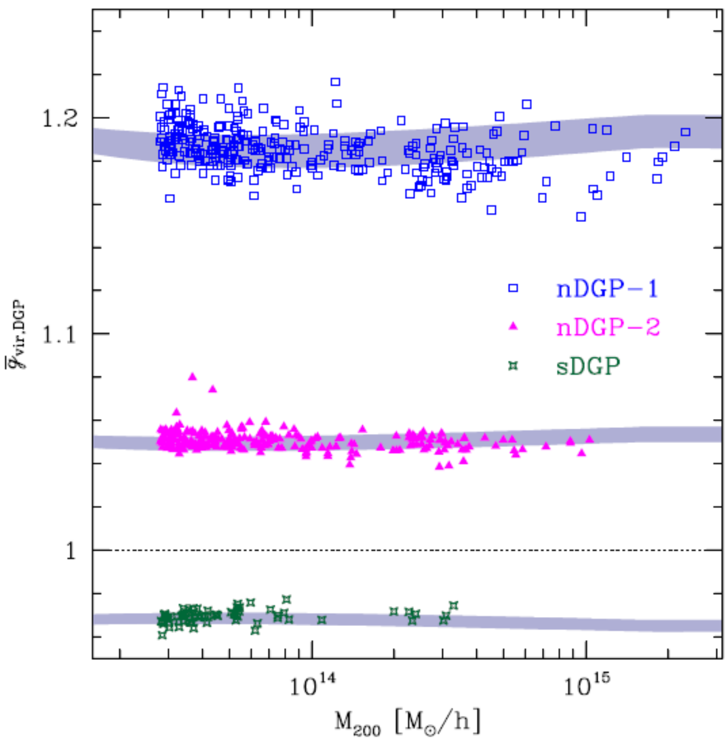
- Vainshtein mechanism

  dark matter halos Schmidt 1003.0409

  Screening is nrealy independent of environment and mass

Modified force

(differnece between lensing and dynamical mass



mass

Observational implication

- Morphology dependence

  The non-linear term vanishes for 1D plane wave

$$\left(\nabla^2\varphi\right)^2 - \partial_i\partial_j\varphi\,\partial^i\partial^j\varphi$$

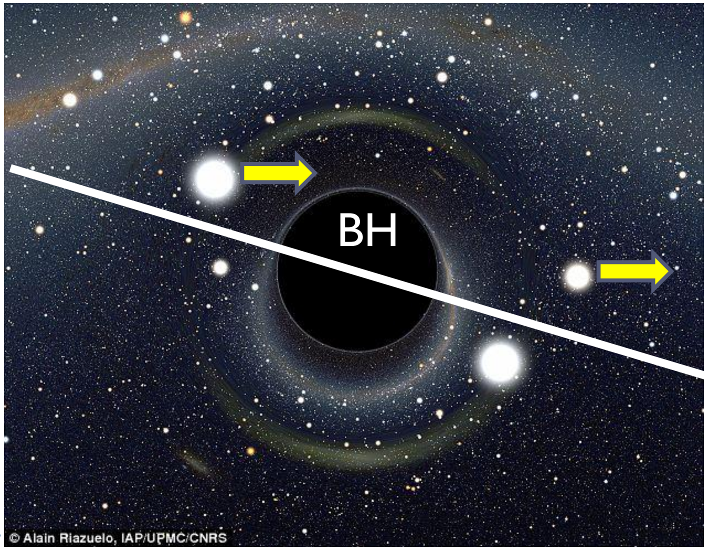
  screening is weak in filaments

- Apparent equivalent principle violation

  Hui, Nicolis 1201.1508

  stars can feel an external field generated by large scale structure but a black hole does not due to no hair theorem

  central BH lag behind stars



$$r = 0.1\,\mathrm{kpc}\left(\frac{2\alpha^2}{1}\right)\left(\frac{|\vec{\nabla}\Phi_{\rm ext}|}{20(\mathrm{km/s})^2/\mathrm{kpc}}\right)\left(\frac{0.01\mathrm{M}_\odot\mathrm{pc}^{-3}}{\rho_0}\right)$$

Vainshtein screened two bodies

- Non-superposition Hiramatsu, Hu, KK, Schmidt

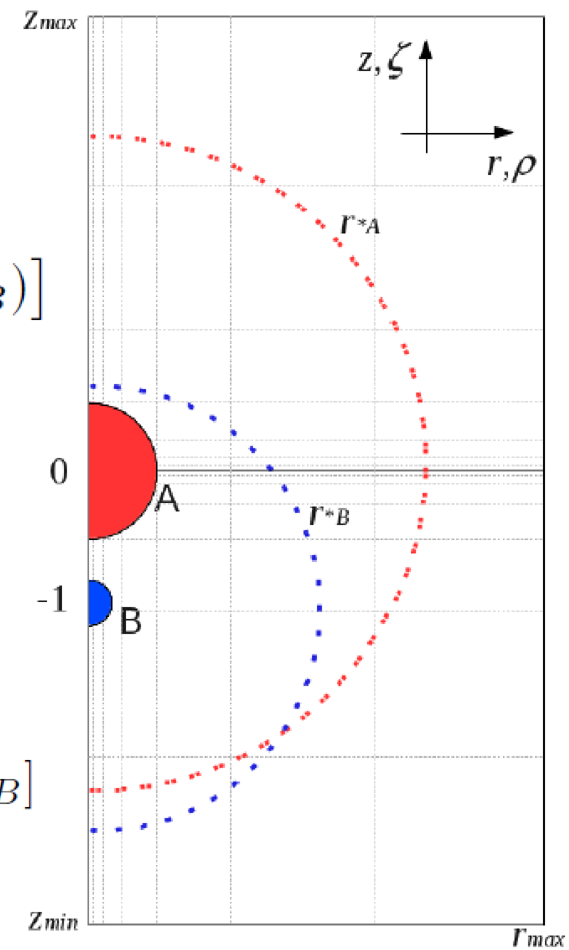
$$\nabla^2\phi = g(a)a^2\left(8\pi G\delta\rho - N[\phi,\phi]\right)$$

$$N[\phi_A,\phi_B] = \frac{r_c^2}{a^4}\left[(\nabla^2\phi_A\nabla^2\phi_B - (\nabla_i\nabla_j\phi_A)(\nabla^i\nabla^j\phi_B)\right]$$

Two body problem (cf. Earth-moon)

$$\phi = \phi_A + \phi_B + \phi_\Delta$$

$$\nabla^2\phi_\Delta + N[\phi_\Delta,\phi_\Delta] + 2N[\phi_A+\phi_B,\phi_\Delta] = -2N[\phi_A,\phi_B]$$



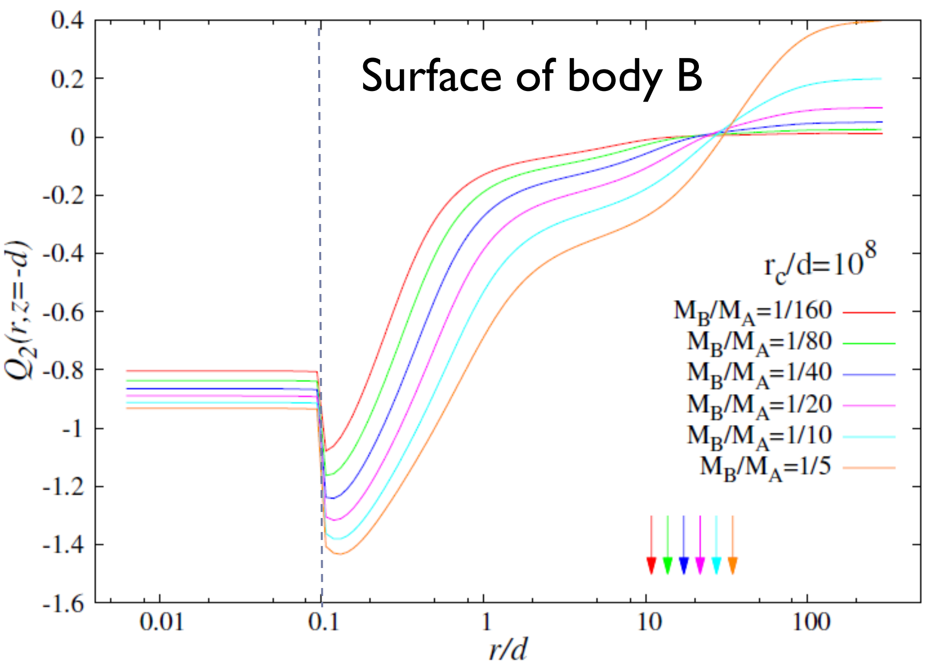
Second derivatives

- Near a small body B (moon)

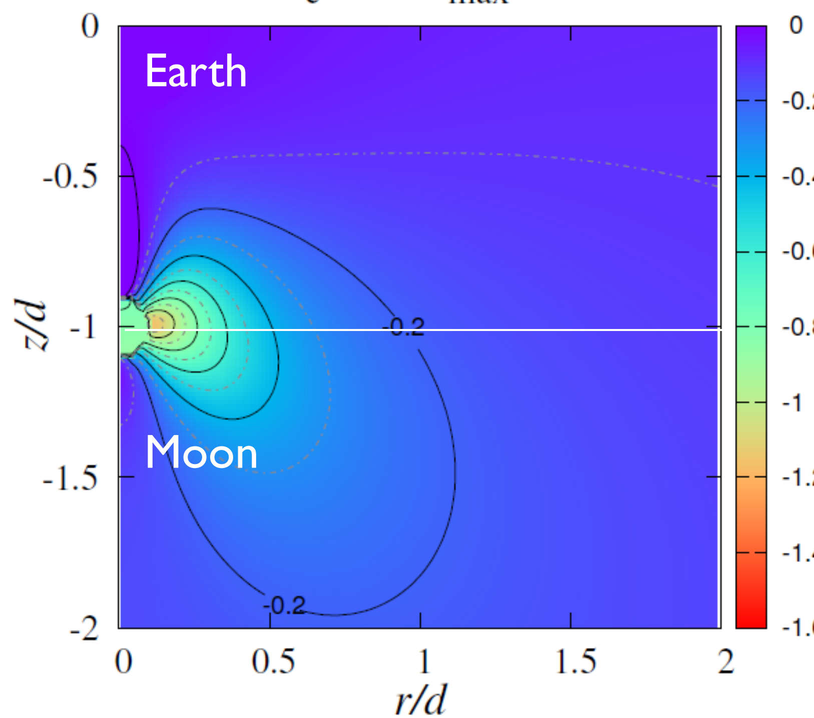
$$\nabla^2 \phi_\Delta = -\nabla^2 \phi_A + O(\sqrt{M_A / M_B})$$

the interference term cancels the second derivative of the field from the large body (Earth)

$$Q_2(r,z) = \frac{\nabla^2 \phi_\Delta}{\nabla^2 \phi_A}$$



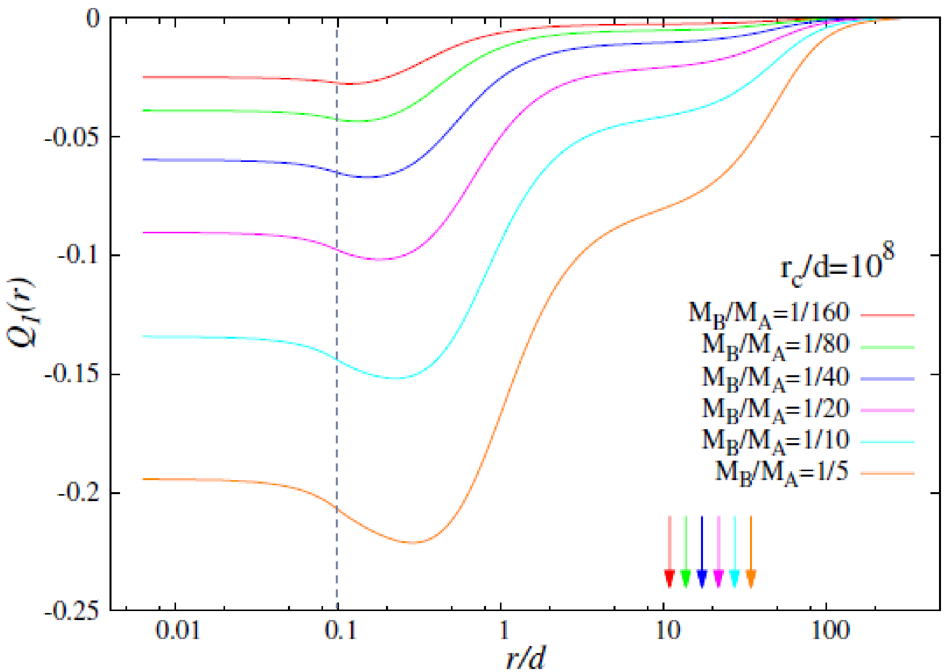
$r_c=10^8$, $r_{max}=286.7$



First derivatives

- Effects on the first derivative (non-radial force) is small

$$Q_1(r) = \frac{\partial_z \phi_\Delta}{\partial_z \phi_A}\Big|_{z=-d}$$



$$Q_1(0) = -0.56\left(\frac{M_B}{M_A}\right)^{0.6}$$

precession anomaly per orbit

$$\frac{\Delta\varphi_{\mathrm{DGP}}}{P} = \frac{3}{8}\frac{1}{r_c}(1+Q_1)$$

$Q_1 \approx 0.04$ for earth-moon

The motion of screened objects depend on their mass

Linerisation

Hui, Nicolis 1201.1508

- If body B is outside of the Vainshtein radius of body A

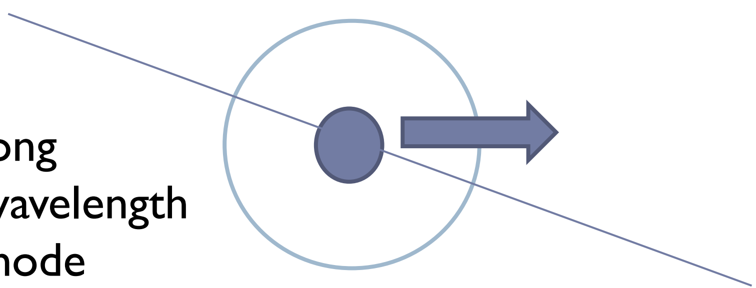
  body B still feels the force from body A as we can add a constant gradient to the solution (Galileon symmetry!)

$$\nabla\varphi = \nabla\varphi_B + \nabla\varphi_A \qquad \nabla\varphi_A \sim \text{const. near body B}$$

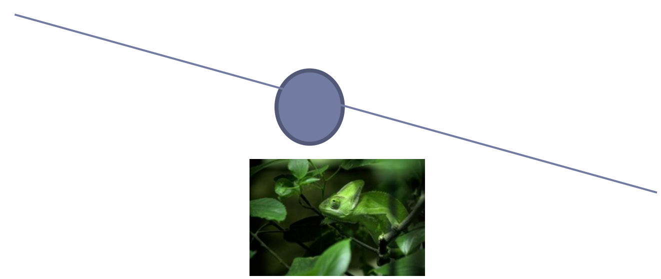
- Two "screening" mechanisms give very different pictures

Vainshtein

long wavelength mode



Chameleon



Challenge for simulations

- Screening mechanism

  governed by a non-linear Poisson equation

  $$\nabla^2\phi = 4\pi G A(\phi)\rho + V'(\phi) + N[\partial\phi, \partial^2\phi]$$

  no superposition rule

  it is not possible to separate long and short range forces

  need to solve the non-linear Poisson equation on a mesh

Oyaizu et.al, Schmidt et.al.

MLAPM	Li, Zhao 0906.3880, Li, Barrow 1005.4231 Zhao, Li, Koyama 1011.1257
ECOSMOG (based on RAMSES)	Li, Zhao, Teyssier, Koyama 1110.1379 Jennings et.al. 1205.2698, Li et.al. 12064317 Brax et.al. 1206.3568

Conclusion

- Modifications to GR

 generally introduce the fifth force, which should be screend

1) break equivalence principle and remove coupling to baryons

 Einstein frame - interacting dark energy models

2) Environmentally (density) dependent screening

 Chameleon/Symmetron/dilaton models

3) Vainshtein mechanism

 massive gravity, Galileon models, braneworld models

Non-linearity of the Poisson equation for the fifth force leads to rich phenomenology