A Proxy for Massive Gravity

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outline

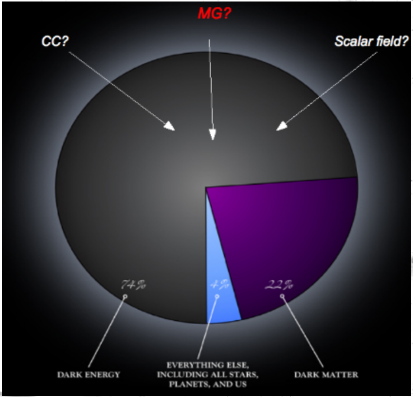
Universe is accelerating, $\Omega_\Lambda \approx 0.72 \pm 0.08$

"The Universe never did make sense; I suspect it was built on government contract". (Robert A. Heinlein)

What is Dark Energy?

3 options?

- **Cosmological Constant**
  (Why is it so small?)
  $\rightarrow$ cosmological constant problem?
- **Dark Energy**
  (Why don't we see them? Similar fine-tuning problem?)
- **Modified gravity**
  (Is there any viable model?)
  $\rightarrow$ massive gravity?



Infra-red Modification of GR

Motivations for IR Modification of GR

- a very nice alternative to the CC or dark energy for explaining the recent acceleration of the Hubble expansion
- a way of attacking the Cosmological Constant problem (fine-tuning problem)
  $\Lambda_{\text{obs}} = \Lambda_{\text{bare}} + \Delta\Lambda \sim (10^{-3}\text{eV})^4$ with $\Delta\Lambda \sim \text{TeV}^4$
- fun!

Modified Gravity

Let's concentrate on the third option: **Modifying gravity**

Maybe not modifying that much! only close to the horizon scale ($\sim 1\text{Gpc}/h$) , corresponding to modifying gravity today (low energy scales).

New degrees of freedom (dof) in the infra-red (IR)

Modifying gravity in the IR typically requires new dof
usually: scalar field

$\mathcal{L} = -\frac{1}{2}\mathcal{Z}_\phi(\partial\delta\phi)^2 - \frac{1}{2}{m_\phi}^2(\delta\phi)^2 - g_\phi\delta\phi T$
where these quantities $\mathcal{Z}_\phi, m_\phi, g_\phi$ depend on the field.

Density dependent mass

- Chameleon
  m_ϕ depends on the environment
  (Khoury, Weltman 2004)

Density dependent coupling

- **Vainshtein** (1971)
  $\mathcal{Z}_\phi$ depends on the environment
- Symmetron
  g_ϕ depends on the environment
  (Hinterbichler, Khoury 2010)

Ghost-free extension of FP = dRGT

a 4D covariant theory of a massive spin-2 field

$$\mathcal{L} = \frac{M_p^2}{2}\sqrt{-g}\left(R - \frac{m^2}{4}\mathcal{U}(g,H)\right)$$

the most generic potential that bears no ghosts is
$\mathcal{U}(g,H) = -4\left(\mathcal{U}_2 + \alpha_3\,\mathcal{U}_3 + \alpha_4\,\mathcal{U}_4\right)$ where the covariant tensor
$H_{\mu\nu} = h_{\mu\nu} + 2\Phi_{\mu\nu} - \eta^{\alpha\beta}\Phi_{\mu\alpha}\Phi_{\beta\nu}$ and the potentials:

$$\begin{aligned}
\mathcal{U}_2 &= [\mathcal{K}]^2 - [\mathcal{K}^2] \\
\mathcal{U}_3 &= [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3] \\
\mathcal{U}_4 &= [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]
\end{aligned}$$

where $\mathcal{K}^\mu_\nu(g,H) = \delta^\mu_\nu - \sqrt{\delta^\mu_\nu - H^\mu_\nu}$, $\Phi_{\mu\nu} = \partial_\mu\partial_\nu\phi$ and $[..]$ =trace.
(de Rham, Gabadadze, Tolley (Phys.Rev.Lett.106,231101))

Decoupling limit (DL)

Decoupling limit
($M_p \to \infty$, $m \to 0$ with $\Lambda_3^3 = m^2 M_p \to$ const)
and decomposition of $H_{\mu\nu}$ in terms of the canonically normalized helicity-2 and helicity-0 fields

$$H_{\mu\nu} = \frac{h_{\mu\nu}}{M_p} + \frac{2\partial_\mu\partial_\nu\phi}{\Lambda_3^3} - \frac{\partial_\mu\partial^\alpha\phi\partial_\nu\partial_\alpha\phi}{\Lambda_3^6}$$

gives the following scalar-tensor interactions

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}_{\mu\nu}{}^{\alpha\beta}h_{\alpha\beta} + h^{\mu\nu}\sum_{n=1}^{3}\frac{a_n}{\Lambda_3^{3(n-1)}}X^{(n)}_{\mu\nu}[\Phi]$$

where $a_1 = -\frac{1}{2}$ and $a_{2,3}$ are two arbitrary constants and $X^{(1,2,3)}_{\mu\nu}$ denote the interactions of the helicity-0 mode

$$X^{(1)}_{\mu\nu} = \Box\phi\eta_{\mu\nu} - \Phi_{\mu\nu}$$

$$X^{(2)}_{\mu\nu} = \Phi^2_{\mu\nu} - \Box\phi\Phi_{\mu\nu} - \frac{1}{2}([\Phi^2] - [\Phi]^2)\eta_{\mu\nu}$$

$$X^{(3)}_{\mu\nu} = 6\Phi^3_{\mu\nu} - 6[\Phi]\Phi^2_{\mu\nu} + 3([\Phi]^2 - [\Phi^2])\Phi_{\mu\nu} - \eta_{\mu\nu}([\Phi]^3 - 3[\Phi^2][\Phi] + 2[\Phi^3])$$

Diagonalized interactions

The transition to Einsteins frame is performed by the change of variable

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - 2a_1\phi\eta_{\mu\nu} + \frac{2a_2}{\Lambda_3^3}\partial_\mu\phi\partial_\nu\phi$$

one recovers Galileon interactions for the helicity-0 mode of the graviton

$$\mathcal{L} = -\frac{1}{2}\bar{h}(\mathcal{E}\bar{h})_{\mu\nu} + 6a_1^2\phi\Box\phi - \frac{6a_2a_1}{\Lambda_3^3}(\partial\phi)^2[\Phi]$$
$$+\frac{2a_2^2}{\Lambda_3^6}(\partial\phi)^2([\Phi^2]-[\Phi]^2) + \frac{a_3}{\Lambda_3^6}h^{\mu\nu}X^{(3)}_{\mu\nu}$$

with the coupling

$$\frac{1}{M_p}\left(\bar{h}_{\mu\nu} - 2a_1\phi\eta_{\mu\nu} + \frac{2a_2}{\Lambda_3^3}\partial_\mu\phi\partial_\nu\phi\right)T^{\mu\nu}$$

Differences to Galileon interactions

Common

- IR modification of gravity as due to a light scalar field with non-linear derivative interactions
- respects the symmetry $\phi(x) \to \phi(x) + c + b_\mu x^\mu$
- Second order equations of motion, containing at most two time derivatives

Different

- undiagonazable interaction $+\frac{a_3}{\Lambda_3^6} h^{\mu\nu} X^{(3)}_{\mu\nu}$ $\to$ important for the self-accelerating solution
- extra coupling $\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}$ $\to$ important for the degravitating solution
- only 2 free-parameters
- **observational difference** due to $\frac{a_3}{\Lambda_3^6} h^{\mu\nu} X^{(3)}_{\mu\nu}$ and $\partial_\mu\phi\partial_\nu\phi T^{\mu\nu}$

Two branches

The Lagrangian in the decoupling limit

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + h^{\mu\nu}\sum_{n=1}^{3}\frac{a_n}{\Lambda_3^{3(n-1)}}X^{(n)}_{\mu\nu}[\Phi] + \frac{1}{M_p}h^{\mu\nu}T_{\mu\nu}$$

Self-accelerating solution

- $T_{\mu\nu} = 0$
- $H \neq 0$

Degravitating solution

- $T_{\mu\nu} \neq 0$
- $H = 0$

Equation of motions

The equation of motions for the helicity-2 mode

$$-\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta}+\sum_{n=1}^{3}\frac{a_n}{\Lambda_3^{3(n-1)}}X^{(n)}_{\mu\nu}[\Phi]=-\frac{1}{M_p}T_{\mu\nu}$$

and for helicity-0 mode

$$\partial_\alpha\partial_\beta h^{\mu\nu}\left(a_1\epsilon_\mu{}^{\alpha\rho\sigma}\epsilon_\nu{}^{\beta}{}_{\rho\sigma}+2\frac{a_2}{\Lambda_3^3}\epsilon_\mu{}^{\alpha\rho\sigma}\epsilon_\nu{}^{\beta\gamma}{}_{\sigma}\Pi_{\rho\gamma}+3\frac{a_3}{\Lambda_3^6}\epsilon_\mu{}^{\alpha\rho\sigma}\epsilon_\nu{}^{\beta\gamma\delta}\Pi_{\rho\gamma}\Pi_{\sigma\delta}\right)=0$$

de Rham, Gabadadze, Heisenberg, Pirtskhalava (Phys.Rev.D 83,103516)

- Gravitons form a condensate whose energy density sources self-acceleration
- Gravitons form a condensate whose energy density compensates the cosmological constant

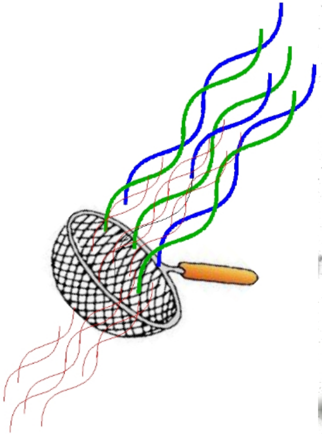
Self-accelerating solution

$$H^2 = m^2 \left(2a_2 q^2 + 2a_3 q^3 - q\right) \text{ and } q = -\frac{a_2}{3a_3} + \frac{(2a_2^2+3a_3)^{1/2}}{3\sqrt{2}a_3}$$

stability

- stable self-accelerating solution: $a_2 < 0$ and $\frac{-2a_2^2}{3} < a_3 < \frac{-a_2^2}{2}$
- $h^{\mu\nu} X^{(3)}_{\mu\nu}$ **plays a crucial role for the stability ($a_3 = 0 \rightarrow$ ghost)**
- kinetic term of the perturbation of the helicity-0 mode survives $\rightarrow$ no strong coupling issues
- no quadratic mixing term between perturbations of the helicity-2 and helicity-0
- cosm. evolution very similar to ΛCDM

Degravitating solution



- degravitating solution: high pass filter modifying the effect of long wavelength sources such as a CC $\rightarrow$ vacuum energy gravitates very weakly
- $H = 0 \quad \rightarrow \quad g_{\mu\nu} = \eta_{\mu\nu}$
- $a_1 q + a_2 q^2 + a^3 q^3 = \frac{-\lambda}{\Lambda_3^3 M_p}$
  as long as the parameter a_3 is present, this equation has always at least one real root
- this static solution is stable for any region of the parameter space for which $2(a_1 + 2a_2 q + 3a_3 q^2) \neq 0$ and real

Proxy theory

We had the following Lagrangian in the decoupling limit

$$\mathcal{L} = -\frac{1}{2}h^{\mu\nu}\mathcal{E}^{\alpha\beta}_{\mu\nu}h_{\alpha\beta} + h^{\mu\nu}X^{(1)}_{\mu\nu} + \frac{a_2}{\Lambda^3}h^{\mu\nu}X^{(2)}_{\mu\nu} + \frac{a_3}{\Lambda^6}h^{\mu\nu}X^{(3)}_{\mu\nu} + \frac{1}{2M_p}h^{\mu\nu}T_{\mu\nu}$$

lets integrate by part the first interaction $h^{\mu\nu}X^{(1)}_{\mu\nu}$:

$$\begin{aligned} h^{\mu\nu}X^{(1)}_{\mu\nu} = h^{\mu\nu}(\Box\phi\eta_{\mu\nu} - \partial_\mu\partial_\nu\phi) &= h^{\mu\nu}(\partial_\alpha\partial^\alpha\phi\eta_{\mu\nu} - \partial_\mu\partial_\nu\phi) \\ &= (\Box h - \partial_\mu\partial_\nu h^{\mu\nu})\phi \\ &= -R\phi \end{aligned}$$

so covariantization of the first interaction: $h^{\mu\nu}X^{(1)}_{\mu\nu} \longleftrightarrow -R\phi$

Proxy theory

Similarly, we can covariantize the other interaction terms. One finds the following correspondences:

$$\begin{aligned} h^{\mu\nu}X^{(1)}_{\mu\nu} &\longleftrightarrow -\phi R \\ h^{\mu\nu}X^{(2)}_{\mu\nu} &\longleftrightarrow -\partial_\mu\phi\partial_\nu\phi G^{\mu\nu} \\ h^{\mu\nu}X^{(3)}_{\mu\nu} &\longleftrightarrow -\partial_\mu\phi\partial_\nu\phi\Phi_{\alpha\beta}L^{\mu\alpha\nu\beta} \end{aligned}$$

such that the Lagrangian becomes

$$\mathcal{L}^\phi = M_p\left(-\phi R - \frac{a_2}{\Lambda^3}\partial_\mu\phi\partial_\nu\phi G^{\mu\nu} - \frac{a_3}{\Lambda^6}\partial_\mu\phi\partial_\nu\phi\Phi_{\alpha\beta}L^{\mu\alpha\nu\beta}\right).$$

with the dual Riemann tensor

$$\begin{aligned} L^{\mu\alpha\nu\beta} = 2R^{\mu\alpha\nu\beta} + 2(R^{\mu\beta}g^{\nu\alpha} + R^{\nu\alpha}g^{\mu\beta} - R^{\mu\nu}g^{\alpha\beta} - R^{\alpha\beta}g^{\mu\nu}) \\ +R(g^{\mu\nu}g^{\alpha\beta} - g^{\mu\beta}g^{\nu\alpha}) \end{aligned}$$

Proxy theory

Instead of focusing on the entire complicated model, study a proxy theory:

$$\mathcal{L} = \sqrt{-g}M_p(M_pR + -\phi R - \frac{a_2}{\Lambda^3}\partial_\mu\phi\partial_\nu\phi G^{\mu\nu} - \frac{a_3}{\Lambda^6}\partial_\mu\phi\partial_\nu\phi\Phi_{\alpha\beta}L^{\mu\alpha\nu\beta})$$

- in 4D $G_{\mu\nu}$ and $L^{\mu\alpha\nu\beta}$ are the only divergenceless tensors $\rightarrow \nabla_\mu G^{\mu\nu} = 0$ and $\nabla_\mu L^{\mu\alpha\nu\beta} = 0$
- All eom are 2^{nd} order $\rightarrow$ No instabilities
- Reproduces the decoupling limit $\rightarrow$ Exhibits the Vainstein mechanism

Chkareuli, Pirtskhalava (Phys.Lett. B713 (2012) 99-103)
de Rham, Heisenberg (PRD84 (2011) 043503)

Self-accelerating solution

- self-acceleration solution: H =const and $\dot{H}=0$.
- make the ansatz $\dot{\phi}=q\frac{\Lambda^3}{H}$.
- assume that we are in a regime where $H\phi \ll \dot{\phi}$

The Friedmann and field equations can be recast in

$$H^2=\frac{m^2}{3}(6q-9a_2q^2-30a_3q^3)$$
$$H^2(18a_2q+54a_3q^2-12)=0$$

Assuming $H\neq 0$, the field equation then imposes,

$$q=\frac{-a_2\pm\sqrt{a_2^2+8a_3}}{6a_3}$$

$\rightarrow$ similar to DL our proxy theory admits a self-accelerated solution, with the Hubble parameter set by the graviton mass.

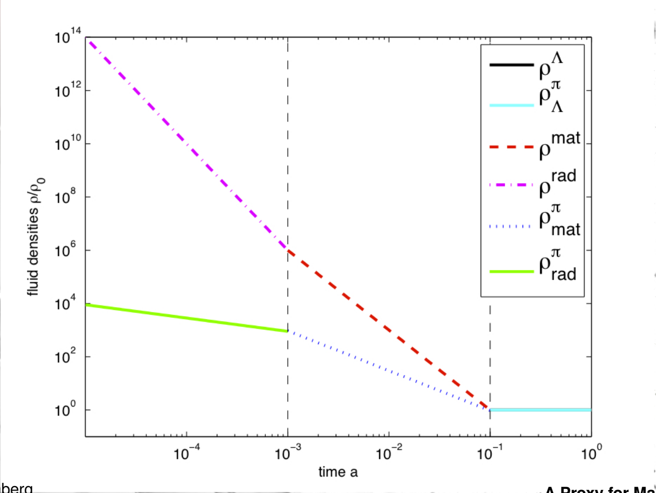
Proxy theory

$$\mathcal{L}^{\phi} = M_p \left(-\phi R - \frac{a_2}{\Lambda^3} \partial_\mu \phi \partial_\nu \phi G^{\mu\nu} - \frac{a_3}{\Lambda^6} \partial_\mu \phi \partial_\nu \phi \Phi_{\alpha\beta} L^{\mu\alpha\nu\beta} \right)$$

- We recover some decoupling limit results:
  - stable self-accelerating solutions within the space parameter space
- During the radiation domination the energy density for ϕ goes as $\rho^{\phi}_{\text{rad}} \sim a^{-1/2}$ and during matter dominations as $\rho^{\phi}_{\text{mat}} \sim a^{-3/2}$ and is constant for later times $\rho^{\phi}_{\Lambda} = \text{const}$
- At early time, interactions for scalar mode are important $\rightarrow$ cosmological screening effect
- Below a critical energy density, screening stop being efficient $\rightarrow$ scalar contribute significantly to the cosmological evolution
- But still the cosmological evolution different than in ΛCDM

Densities

$$\mathcal{L}^\phi = M_p\left(-\phi R - \frac{a_2}{\Lambda^3}\partial_\mu\phi\partial_\nu\phi G^{\mu\nu} - \frac{a_3}{\Lambda^6}\partial_\mu\phi\partial_\nu\phi\Phi_{\alpha\beta}L^{\mu\alpha\nu\beta}\right)$$



Degravitation solution

The effective energy density of the field ϕ is

$$\rho^{\phi} = M_p(6H\dot{\phi} + 6H^2\phi - \frac{9a_2}{\Lambda^3}H^2\dot{\phi}^2 - \frac{30a_3}{\Lambda^6}H^3\dot{\phi}^3)$$

- If one takes $\phi = \phi(t)$ and $H = 0 \to \rho^{\phi} = 0$
  $\to$ so the field has absolutely no effect and cannot help the background to degravitate.
- Fab Four has similar interactions, they find degravitation solution! (arXiv:1208.3373)
  BUT they rely strongly on spatial curvature
- in the absence of spatial curvature $\kappa = 0$, the contribution from the scalar field vanishes if $H = 0$.
- $\to$ BUT relying on spatial curvature brings concerns over instabilities

Conclusion

- decoupling limit of dRGT
  - stable self-accelerating solution similar to ΛCDM
  - degravitating solution
- Proxy theory
  - stable self accelerating solution
  - no degravitating solution
  - the scalar mode does not decouple around the self-accelerating background
  - leads to an extra force during the history of the Universe
  - would influence the time sequence of gravitational clustering and the evolution of peculiar velocities, as well as the number density of collapsed objects.

cosmological observations in Proxy Theory

two categories: measurement of

geometrical probes

the Hubble function

- distance-redshift relation of supernovae
- measurements of the angular diameter distance as a function of redshift (CMB+BAO)

structure formation probes

the Growth function

- homogeneous growth of the cosmic structure
  $\rightarrow$ ISW
- non-linear growth
  $\rightarrow$ gravitational lensing
  $\rightarrow$ formation of galaxies
  $\rightarrow$ clusters of galaxies by gravitational collapse

going on projects with Claudia de Rham, Matthias Bartelmann, Bjoern Malte Schaefer, Rampei Kimura, Jose Beltran Jimenez