INFLATION FROM MAGNETIC DRIFT

Mark Wyman

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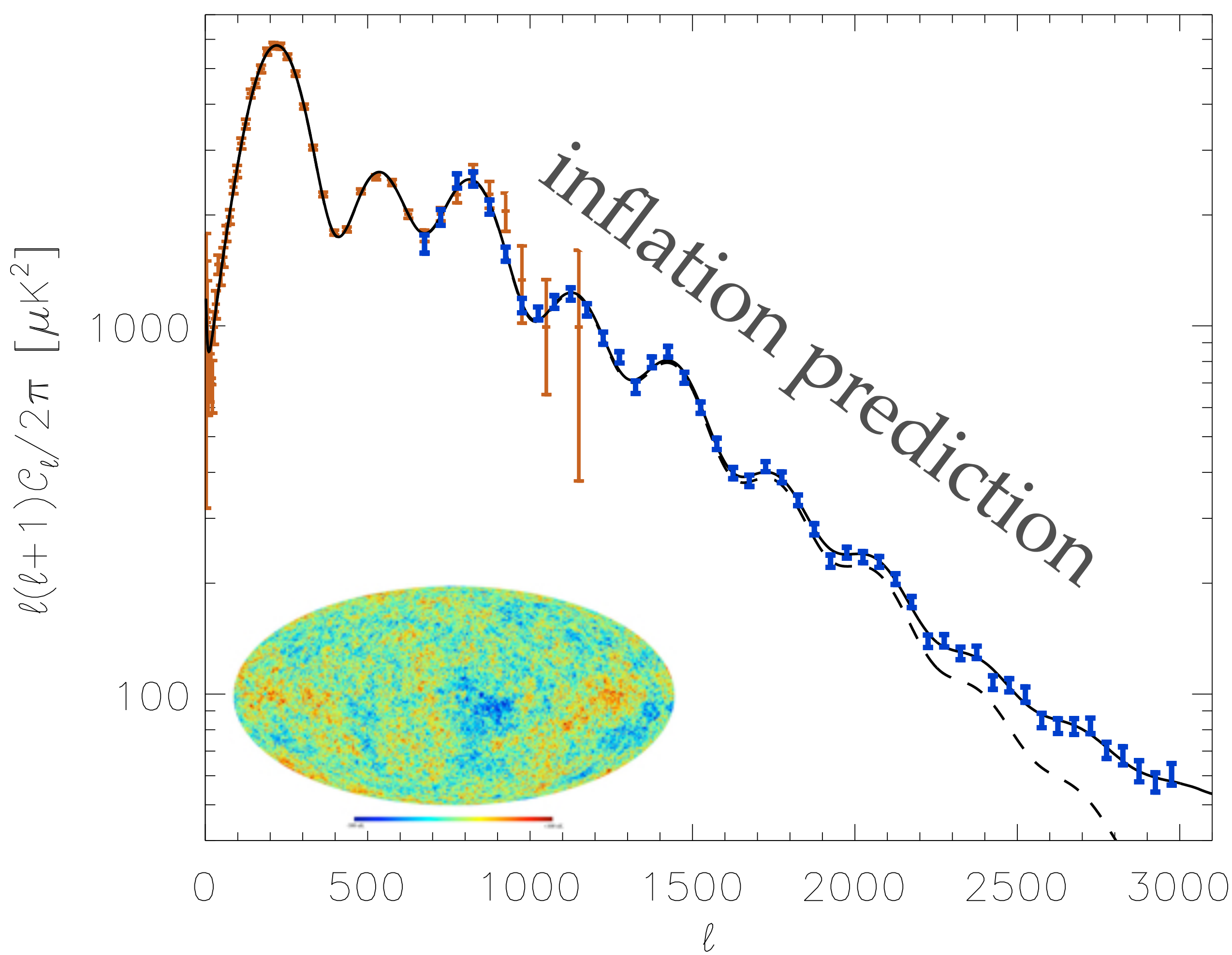
Workshop on Cosmic Acceleration

Carnegie Mellon University, Aug. 25

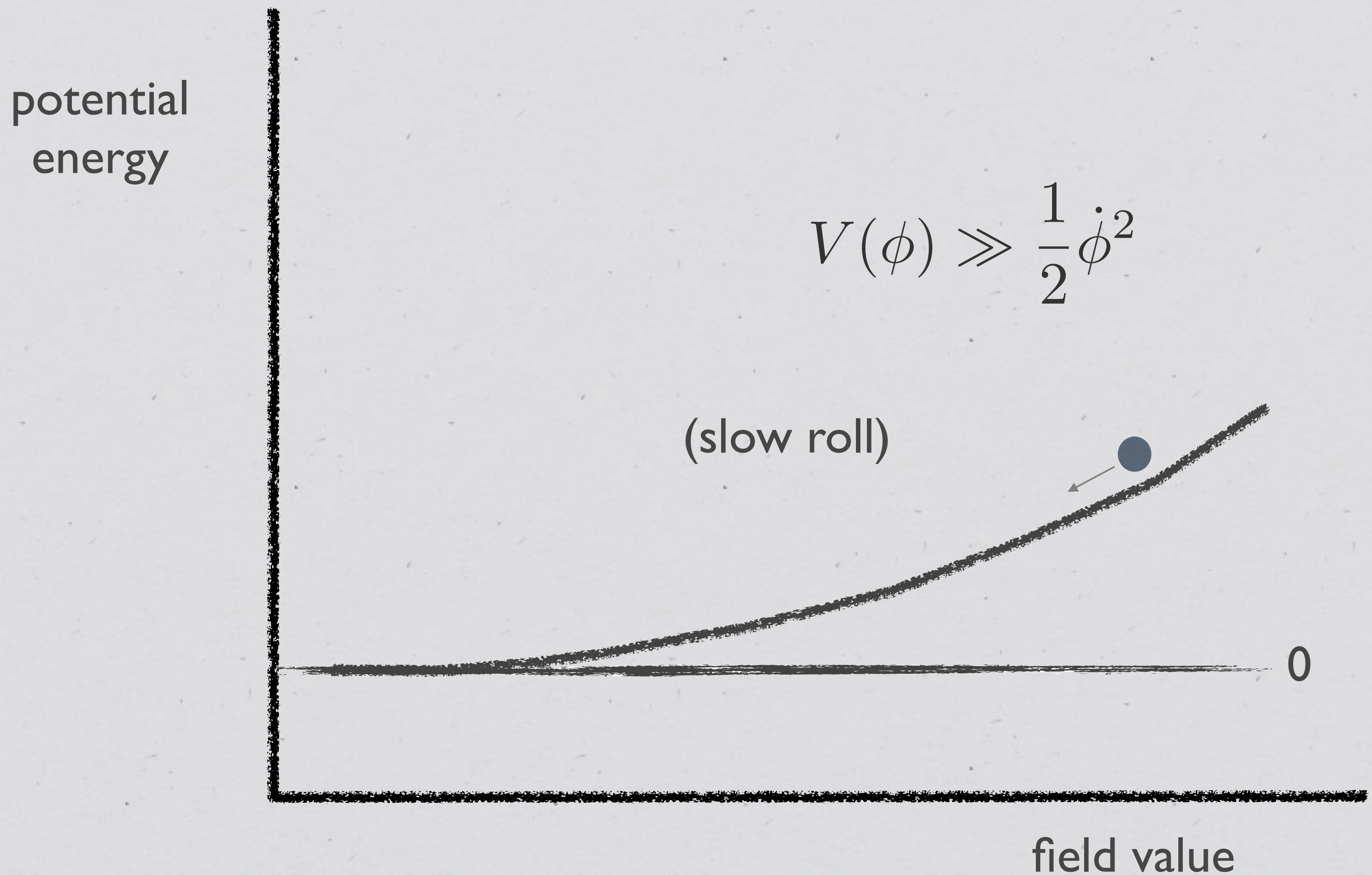
Inflation matches the data.



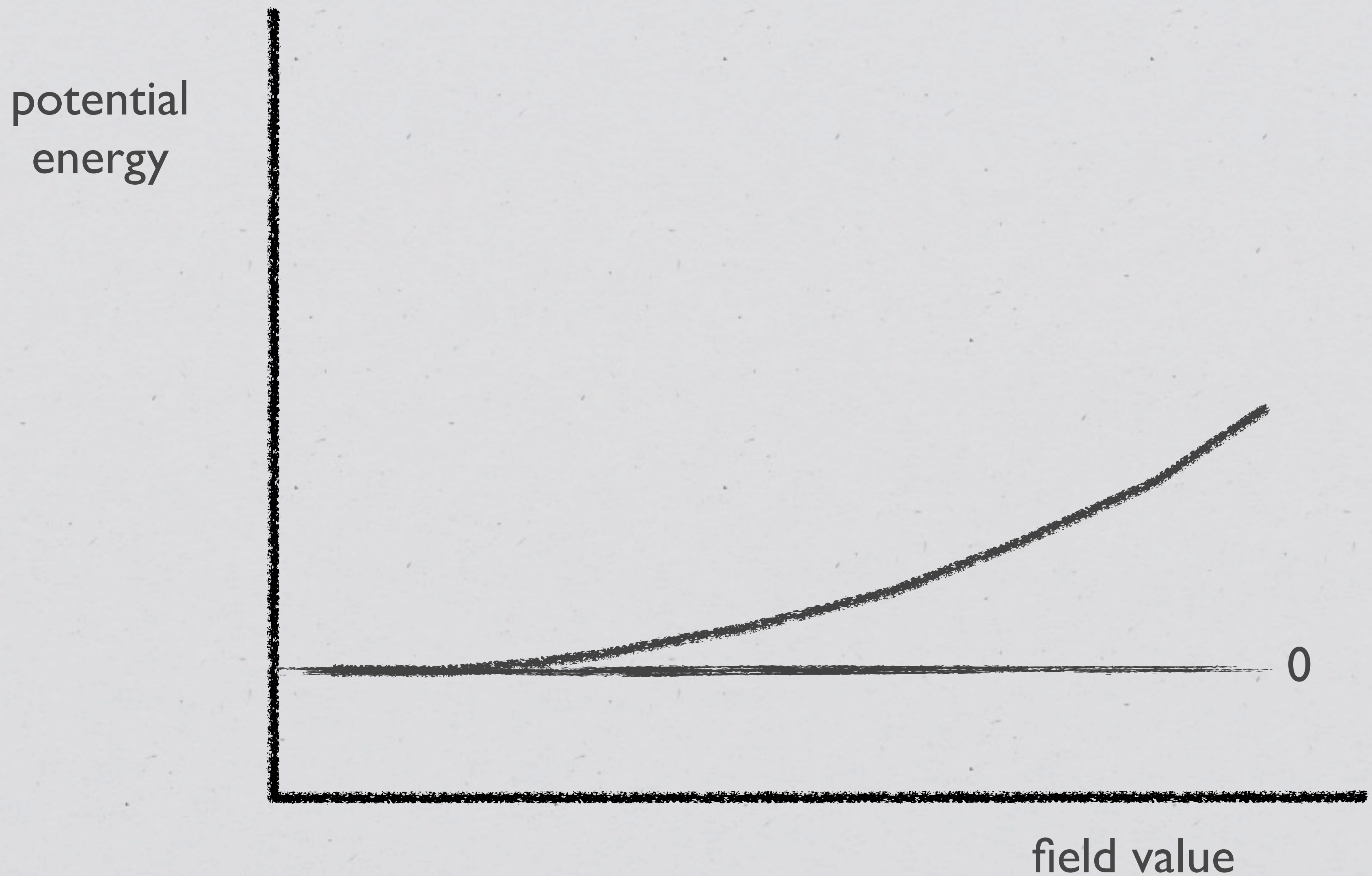
Requirements:

1. Enough inflation
2. Perturbations almost scale invariant
    - A little bigger on long length scales (i.e., red tilt)
3. Correct amplitude of perturbations

Usually, we demand a flat potential.



But flatness isn't safe...



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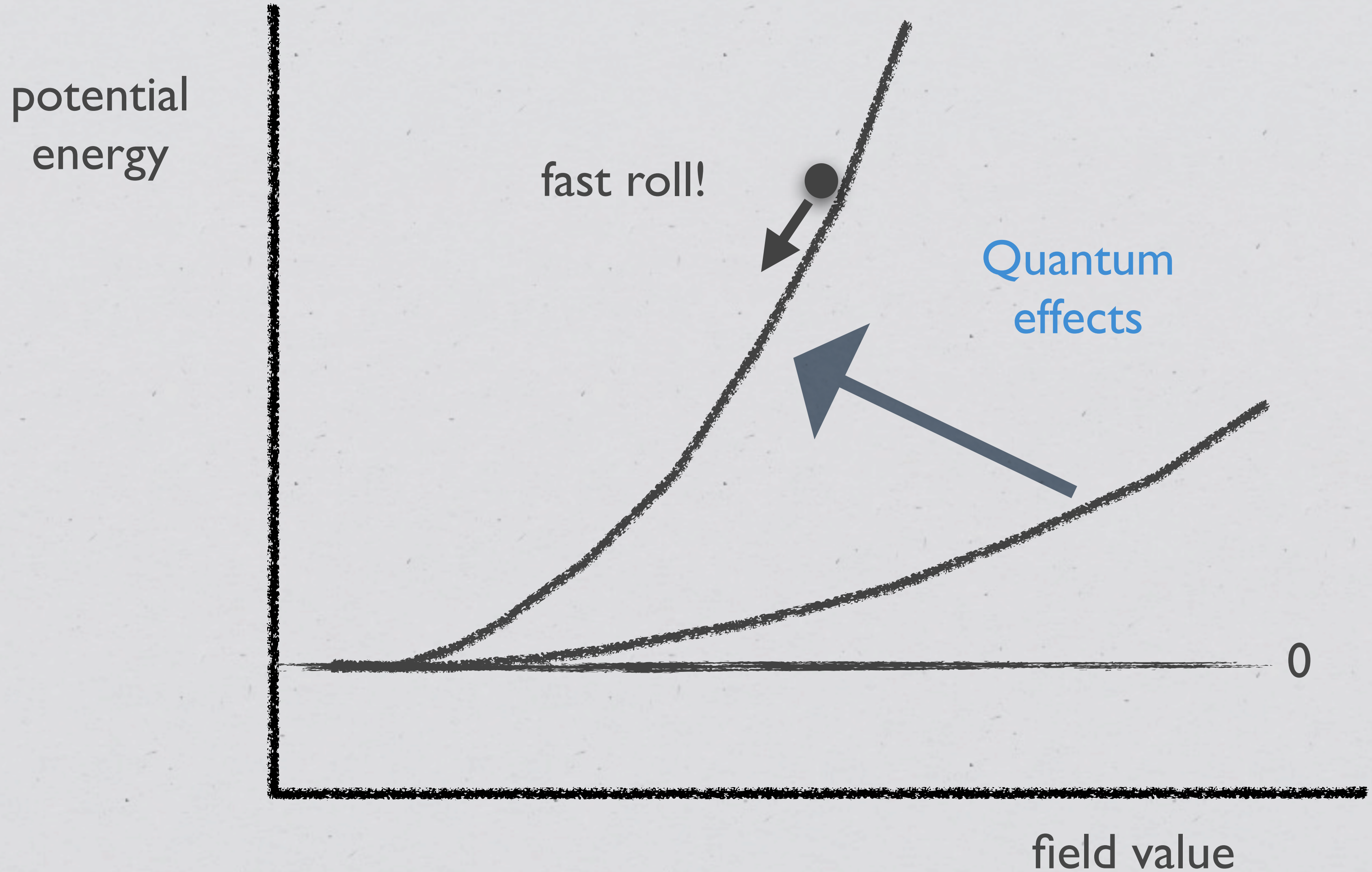

A hierarchy problem

Need to keep $V(\varphi)$ big,

but its derivatives small.

Simplified example theory:

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(\partial\psi)^2 - m^2\phi^2 - M^2\psi^2 - g\phi^2\psi^2 - \mathcal{O}\left(\frac{\psi^4}{M_{pl}^2}\phi^2\right)$$

$m \sim 10^{-4} M_{pl}$ $\quad$ $M \gg m$ $\quad$ $g \sim 1$

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$m \sim 10^{-4} M_{pl}$ $\quad$ $M \gg m$ $\quad$ $g \sim 1$

$$\psi \simeq 0 \rightarrow H \sim \frac{1}{\sqrt{6}}\frac{m\phi}{M_{pl}}$$

$$\phi \sim 14 M_{pl}$$

The "eta" problem

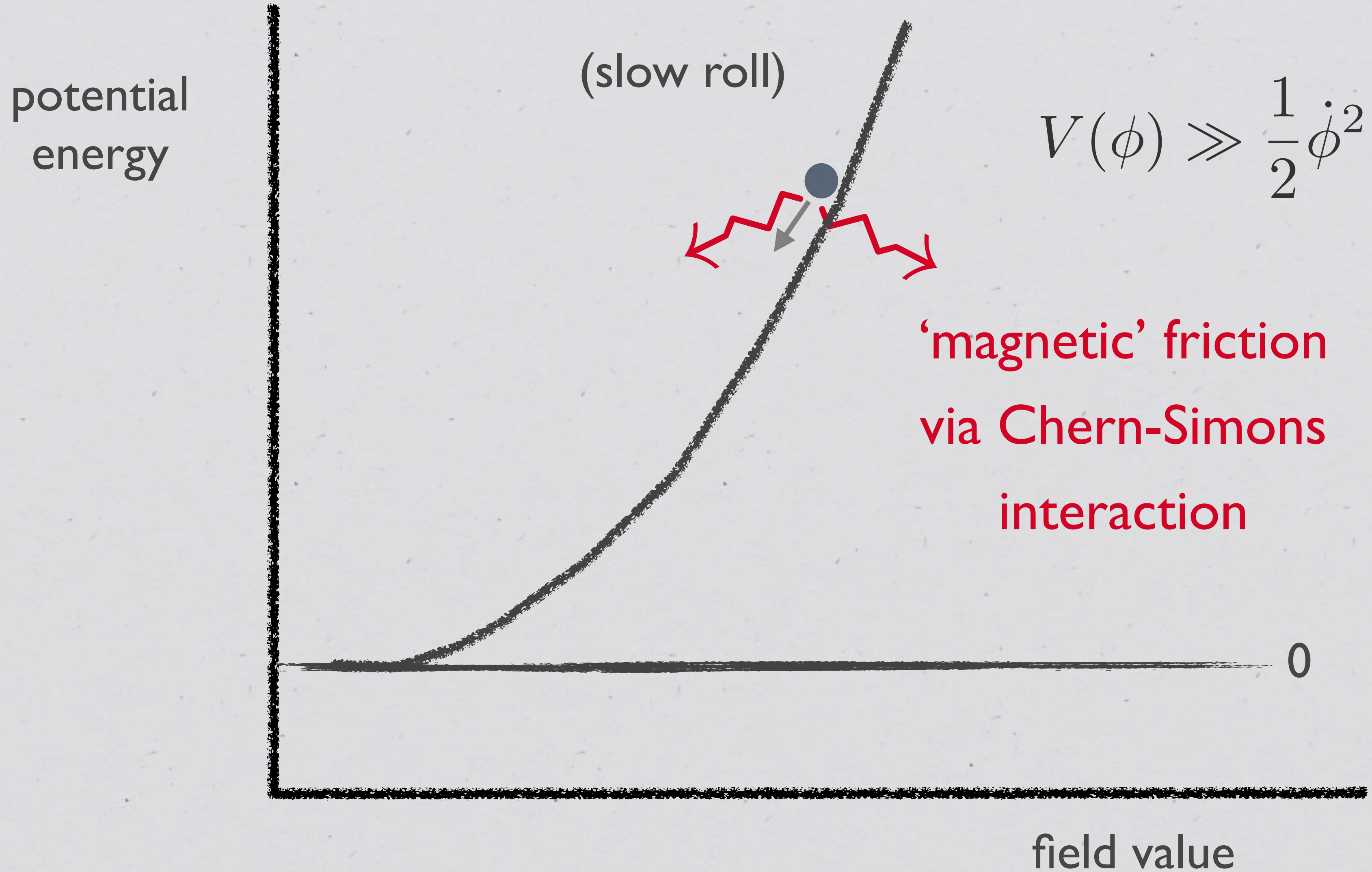
Quantum effects: $\langle\psi^2\rangle \sim H^2 \sim V(\phi)$

$$V_{eff}(\phi) \sim (m^2 + g\,H^2 + \frac{H^4}{M_{pl}^2} + \cdots)\phi^2$$

$$\eta \equiv \frac{V''}{V} = \frac{m^2}{V} \sim 1$$

Too Big!

Today: another route to slow roll

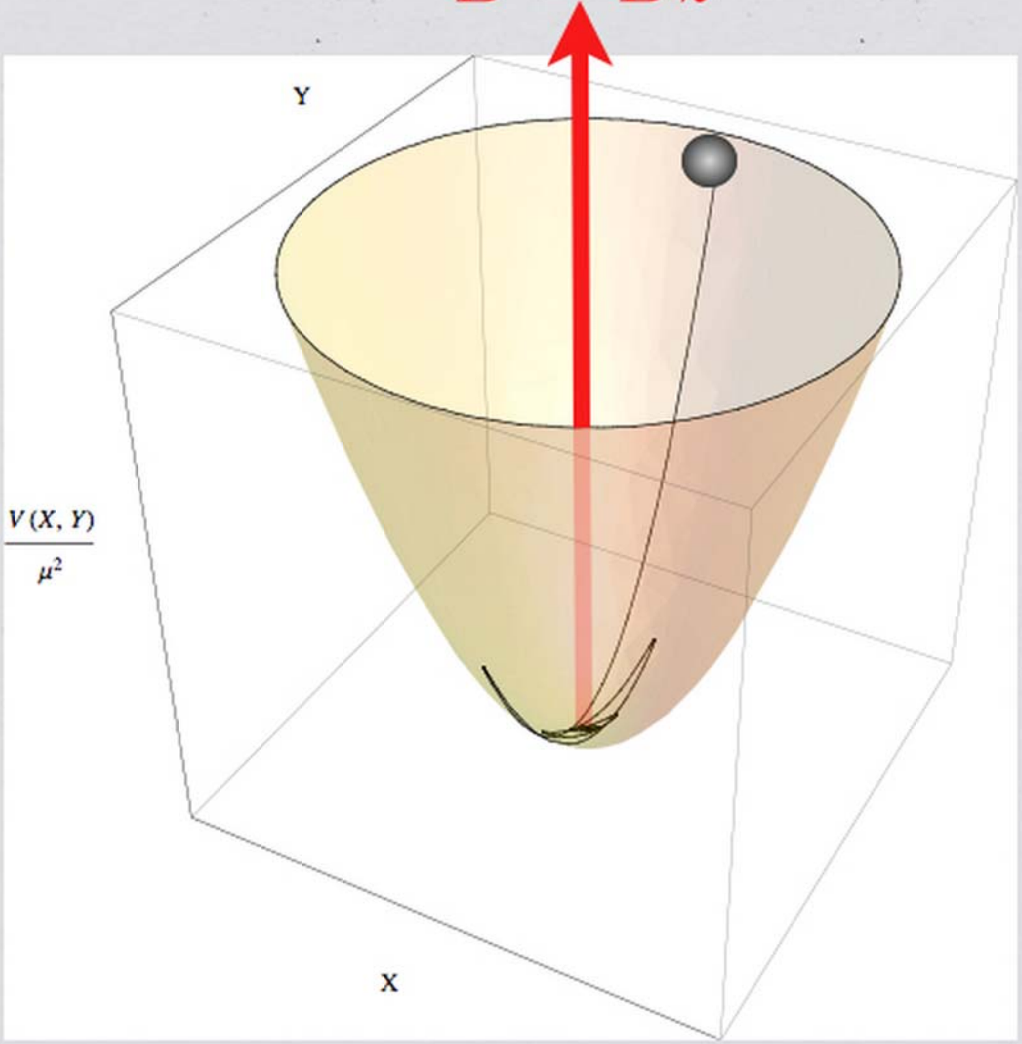


$$V(\phi) \gg \frac{1}{2}\dot{\phi}^2$$

'magnetic' friction via Chern-Simons interaction

An analogy: Magnetic Drift

$\vec{B} = B\hat{z}$



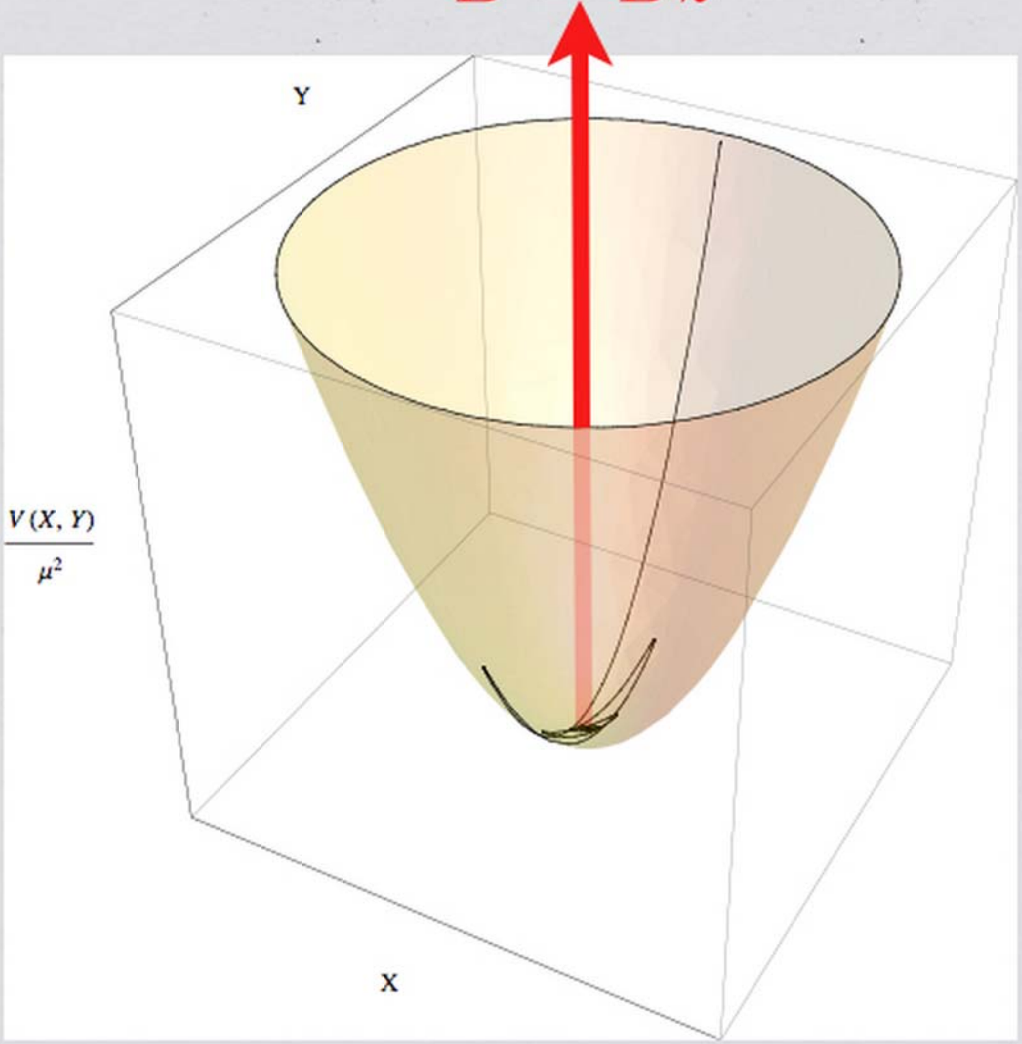
potential force

$$\ddot{X} + H\dot{X} + \mu^2 X = B\dot{Y}$$
$$\ddot{Y} + H\dot{Y} + \mu^2 Y = -B\dot{X}$$

ordinary friction

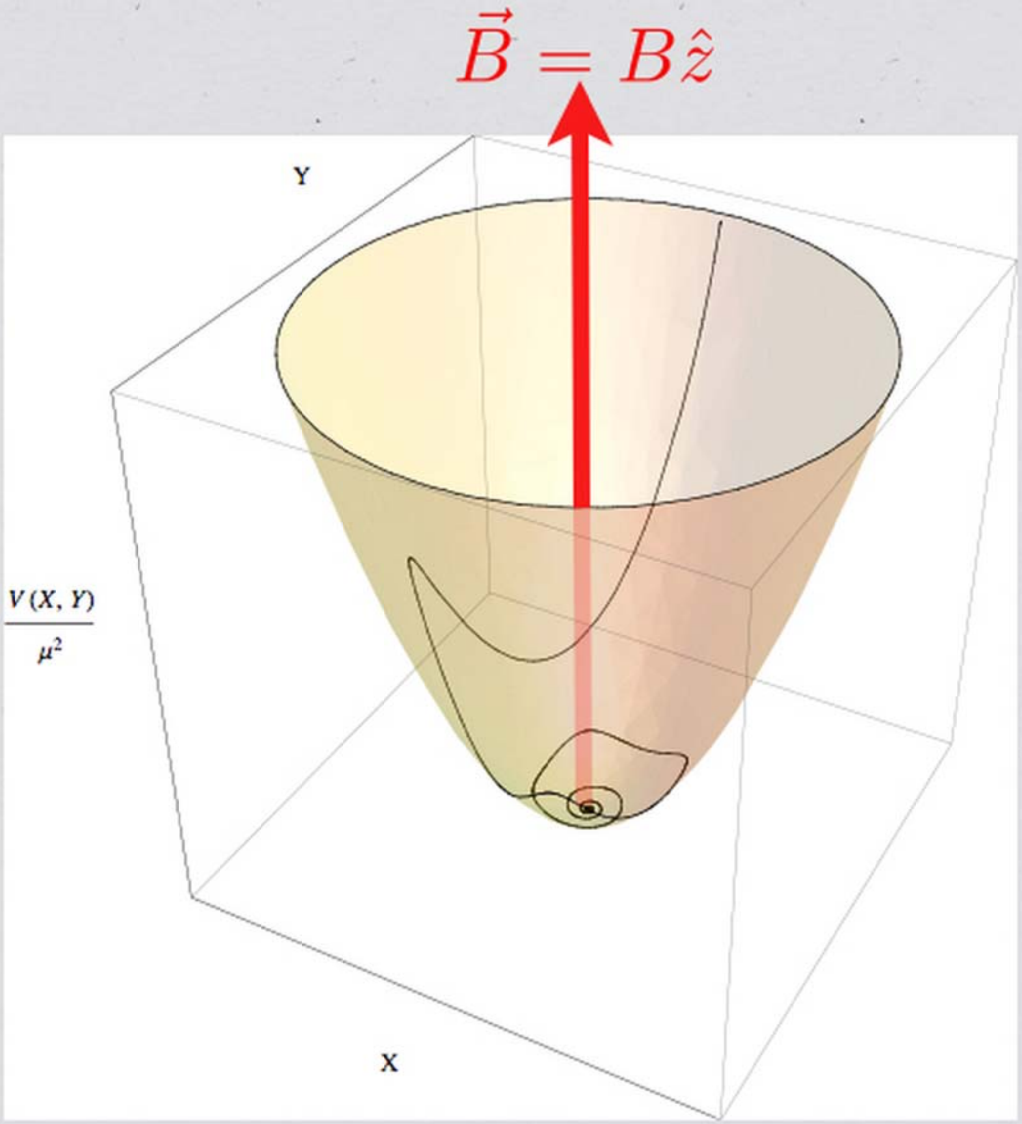
magnetic force

$$\vec{B} = B\hat{z}$$



$$B = 0.1\mu$$

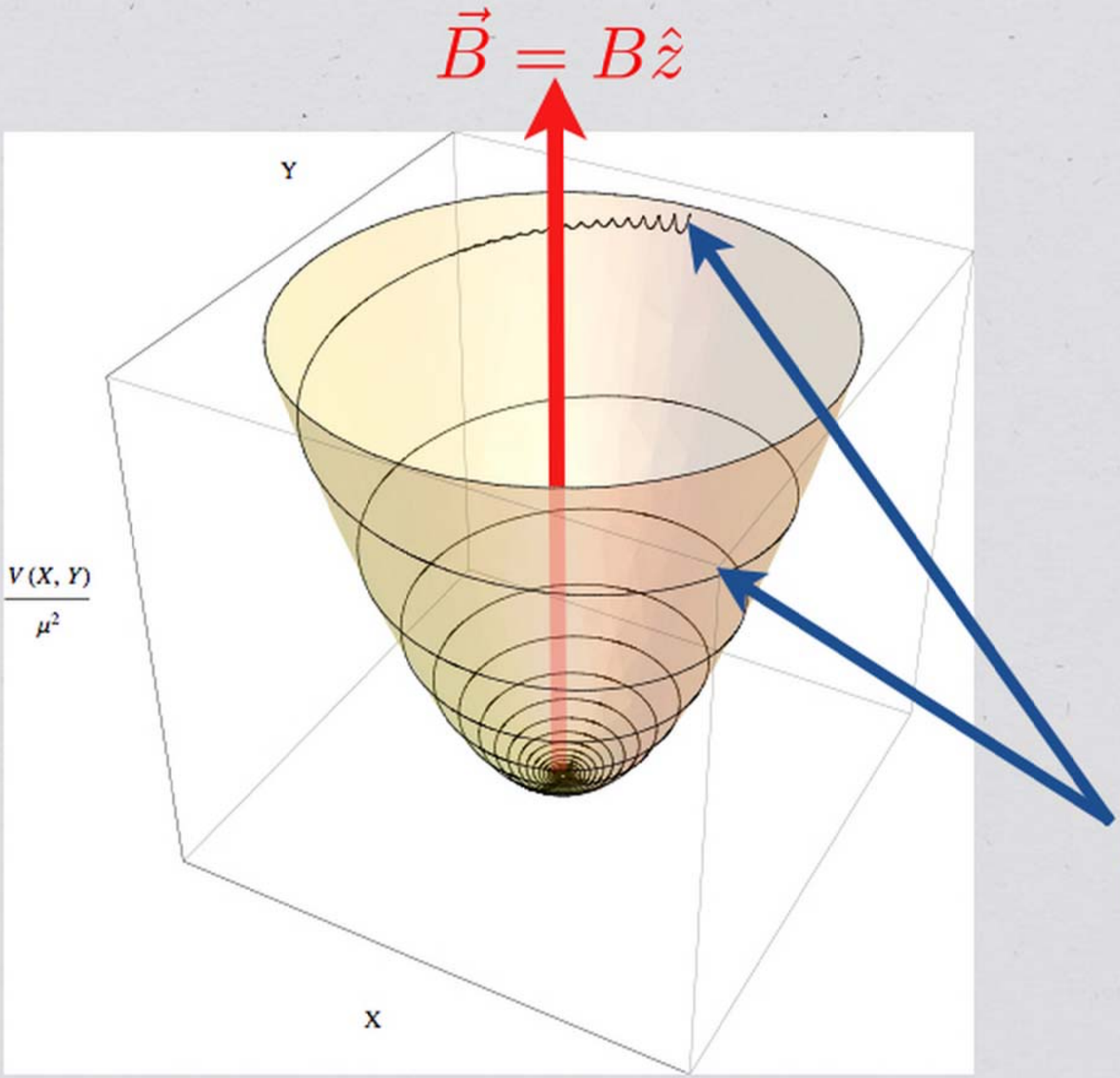
$$H = \frac{\mu^2}{\sqrt{3}}$$



$$B = \mu$$

$$H = \frac{\mu^2}{\sqrt{3}}$$

Magnetic Drift



$$B = 10\mu$$

$$H = \frac{\mu^2}{\sqrt{3}}$$

Fast mode rapidly damped leaving slow magnetic drift mode.

Long **slow** spiral down the potential!

Chern-Simons interactions

$$\mathcal{L} \subset \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu}$$

In E&M, $= \vec{B} \cdot \vec{E}$

Chern-Simons interactions

Facts: $\epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} \longrightarrow$ total derivative

$\epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} \longrightarrow$ no stress-energy

So,

$$\mathcal{X} \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} \to (\mathcal{X} + c) \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu} \to \mathcal{X} \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu}$$

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1. respects a shift symmetry!

Chern-Simons interactions

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1. respects a shift symmetry!
2. transfers energy between scalar and gauge fields

Chern-Simons interactions

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So,

$$\mathcal{X}\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu} \rightarrow (\mathcal{X}+c)\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu} \rightarrow \mathcal{X}\epsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}$$

1. respects a shift symmetry!
2. transfers energy between scalar and gauge fields
3. contributes no stress energy

Key: single time derivative

$$\varepsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta} = d[\text{Something}]$$

integrate action by parts

$$\mathcal{L} \subset \dot{\mathcal{X}}[\text{Something}]$$

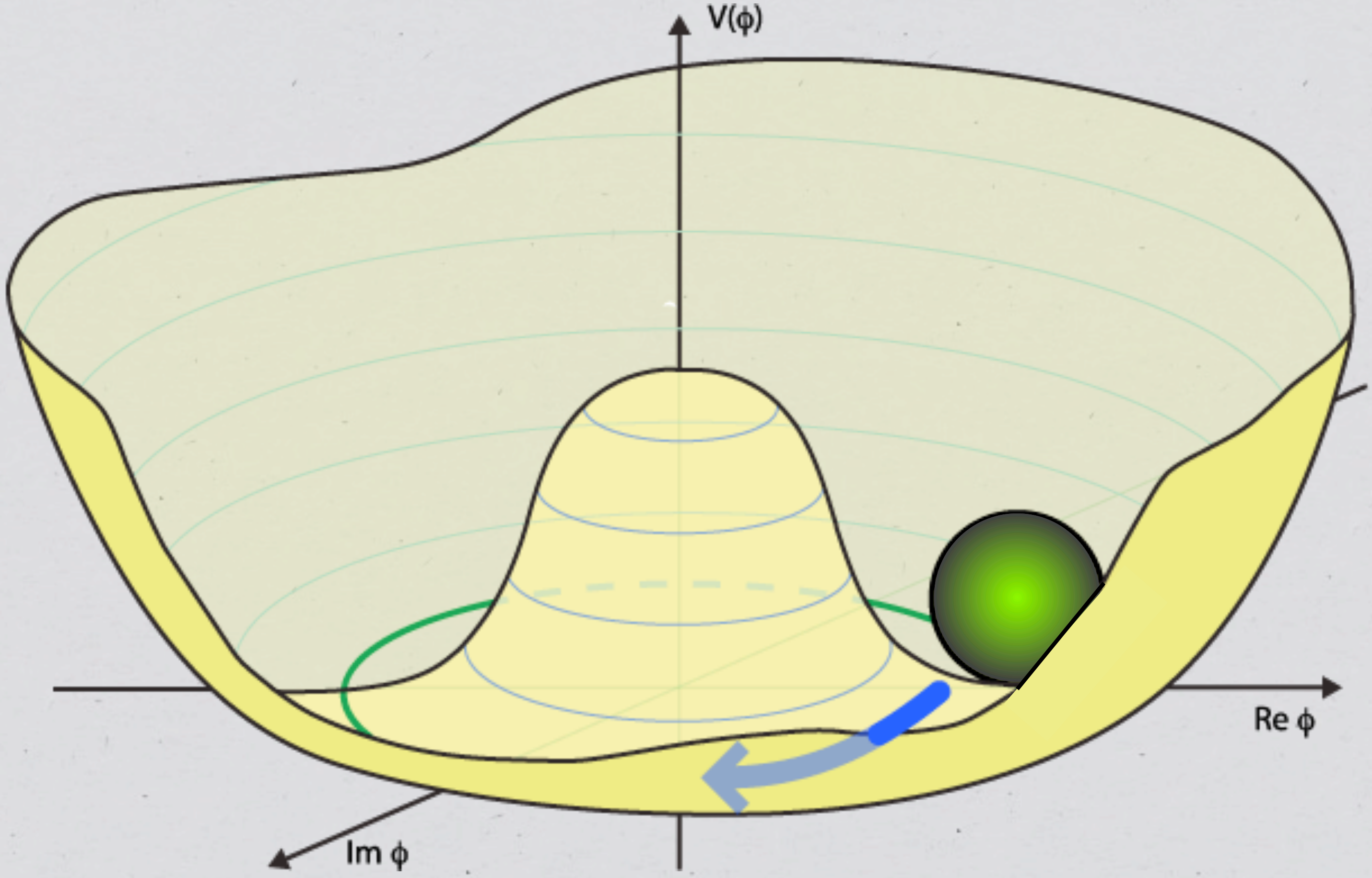
Like Lorentz force! $(\dot{x} \times B)$

Idea: turn inflaton rolling energy into gauge field energy

$$\ddot{\mathcal{X}} + 3H\dot{\mathcal{X}} - V'(\mathcal{X}) = \epsilon^{\alpha\beta\mu\nu} F_{\alpha\beta} F_{\mu\nu}$$

balance these

Axion as inflaton: shift-symmetric pseudo-scalar



$$\mathcal{X} \to \mathcal{X} + c$$

Axion potential:

$$V(\mathcal{X}) = [\text{almost anything works}]$$

- For concreteness, choose:

$$V(\mathcal{X}) = \mu^4 \left(1 + \cos\left(\frac{\mathcal{X}}{f}\right)\right)$$

Natural Inflation: Freese, Frieman, and Olinto (1990)

Next: which F?

First thought: why not Electromagnetism?

Anber & Sorbo (2009):
Naturally inflating on steep potentials through electromagnetic dissipation

Problems:

- Need:
  (quantum photon emission) to **balance** (classical axion evolution)

→ Correct pert. amplitude requires 10^5 gauge fields

→ Blue (not red) spectrum

A solution: **classical**, non-Abelian gauge fields

Discovered ca. 1980:

$$A_0^a = 0 \qquad A_i^a = \psi(t)a(t)\delta_i^a$$

solves the non-Abelian gauge field
equations of motion on an FRW background.

$$E_{\rm chromo} \propto \dot{\psi} + H\psi \qquad\qquad B_{\rm chromo} \propto \tilde{g}\psi^2$$

spatially homogenous, classical fields!

Why does this work?

SU(2) ⟷ SO(3)

simplest non-abelian Lie group → SU(2)

spatial rotations → SO(3)

3 gauge fields ⟷ 3 spatial dimensions

Chromo-Natural Inflation

$$\ddot{\mathcal{X}} + 3H\dot{\mathcal{X}} - V'(\mathcal{X}) = \lambda\epsilon^{\alpha\beta\mu\nu}\delta_{ab}F^a_{\alpha\beta}F^b_{\mu\nu}$$

$$\epsilon^{\alpha\beta\mu\nu}\delta_{ab}F^a_{\alpha\beta}F^b_{\mu\nu} = E \cdot B = -3\tilde{g}\psi^2(\dot{\psi} + H\psi)$$

when $\dot{\psi} \simeq 0$, need $3\lambda\tilde{g}H\psi^3 \simeq V'(\mathcal{X})$

Magnetic Drift

$$\left(3H + \frac{g^2\lambda^2\psi^4}{Hf^2}\right)\dot{\mathcal{X}} = \frac{\mu^4}{f}\sin(\mathcal{X}/f) - \frac{g\lambda}{f}H\psi^3 + \frac{2g^3\lambda}{fH}\psi^5$$

$$\left(3H + \frac{g^2\lambda^2\psi^4}{Hf^2}\right)\dot{\psi} = -2H^2\psi - 2g^2\psi^3 - \frac{g^2\lambda^2}{f^2}\psi^5 + \frac{g\lambda}{3Hf^2}\psi^2\mu^4\sin(\mathcal{X}/f)$$

'magnetic friction'

Dynamics dominated by magnetic drift for: $\frac{\lambda g}{f}\psi^2 \gg \sqrt{3}H$

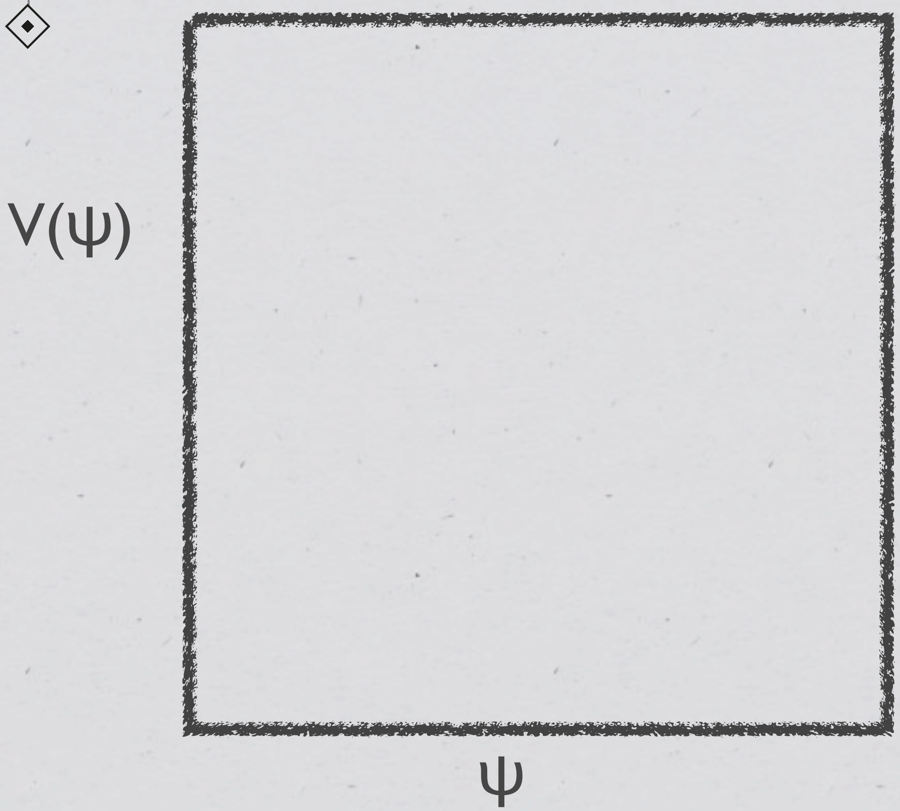
$\dot{\psi} \propto \mathcal{O}(1)$ ⟶ Fast evolution to fixed point

$\dot{\mathcal{X}} \propto \mathcal{O}(1/\lambda)$ ⟶ Slow magnetic drift

Recall: Chern-Simons Term is topological.

⟶ No contribution to $T^{\mu\nu}$

Fixed point for ψ



$$V_{\text{eff}}(\psi) = H^2 \frac{\psi^2}{2} + \frac{\mu^4 \sin\left(\mathcal{X}/f\right)}{3\tilde{g}\lambda} \frac{H}{\psi},$$

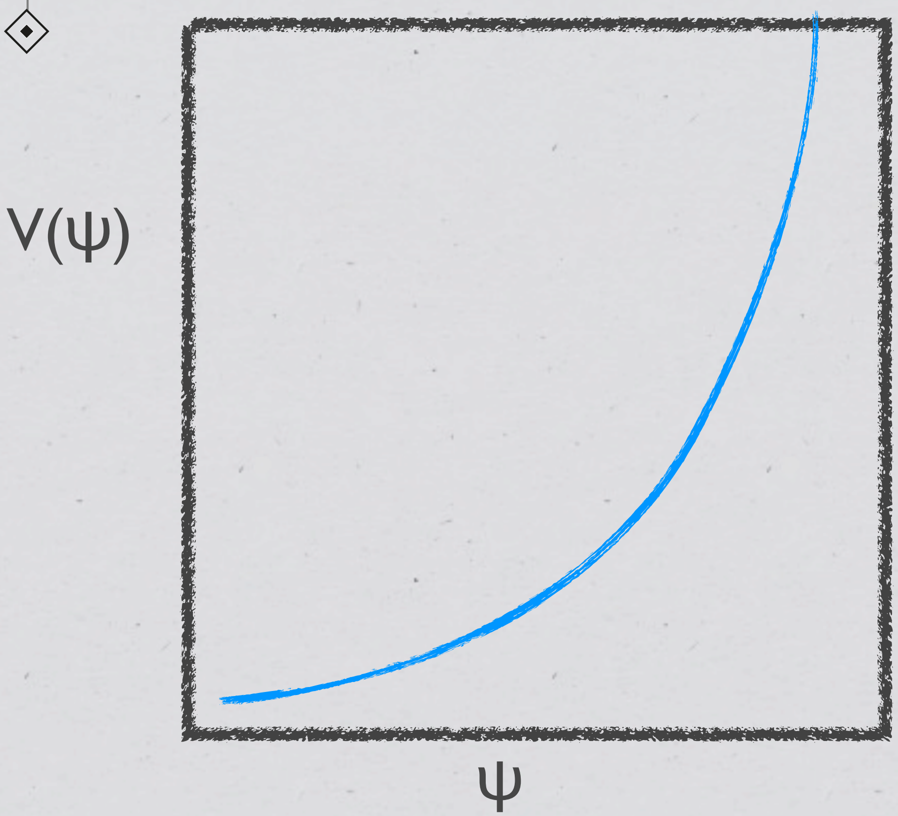
-Fixed point for ψ

$$\psi = \left(\frac{V'}{3g\lambda H} \right)^{1/3}$$

Resulting axion trajectory: $\frac{\dot{\mathcal{X}}}{f} \sim \frac{H^2}{\lambda}$

Velocity independent of V' !

Fixed point for ψ



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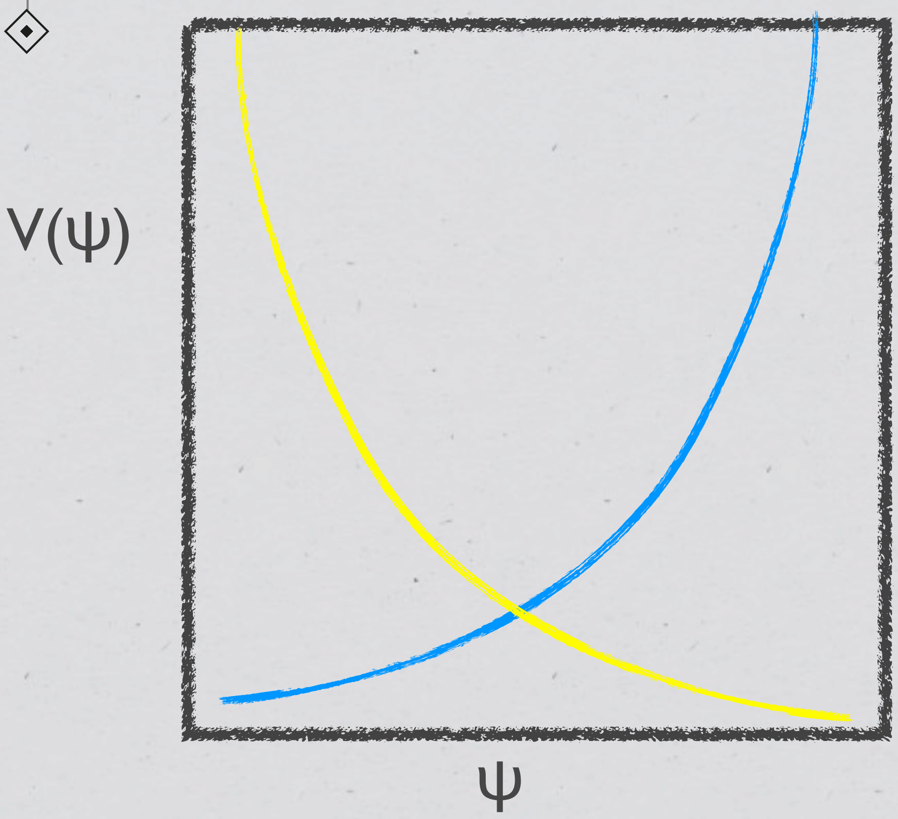

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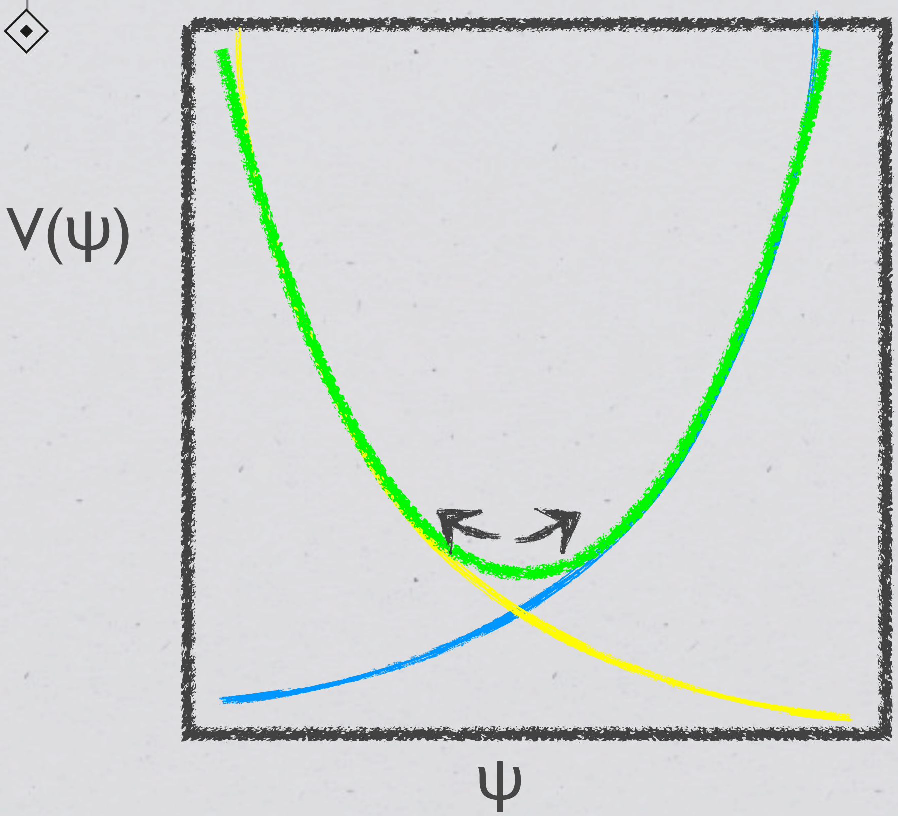

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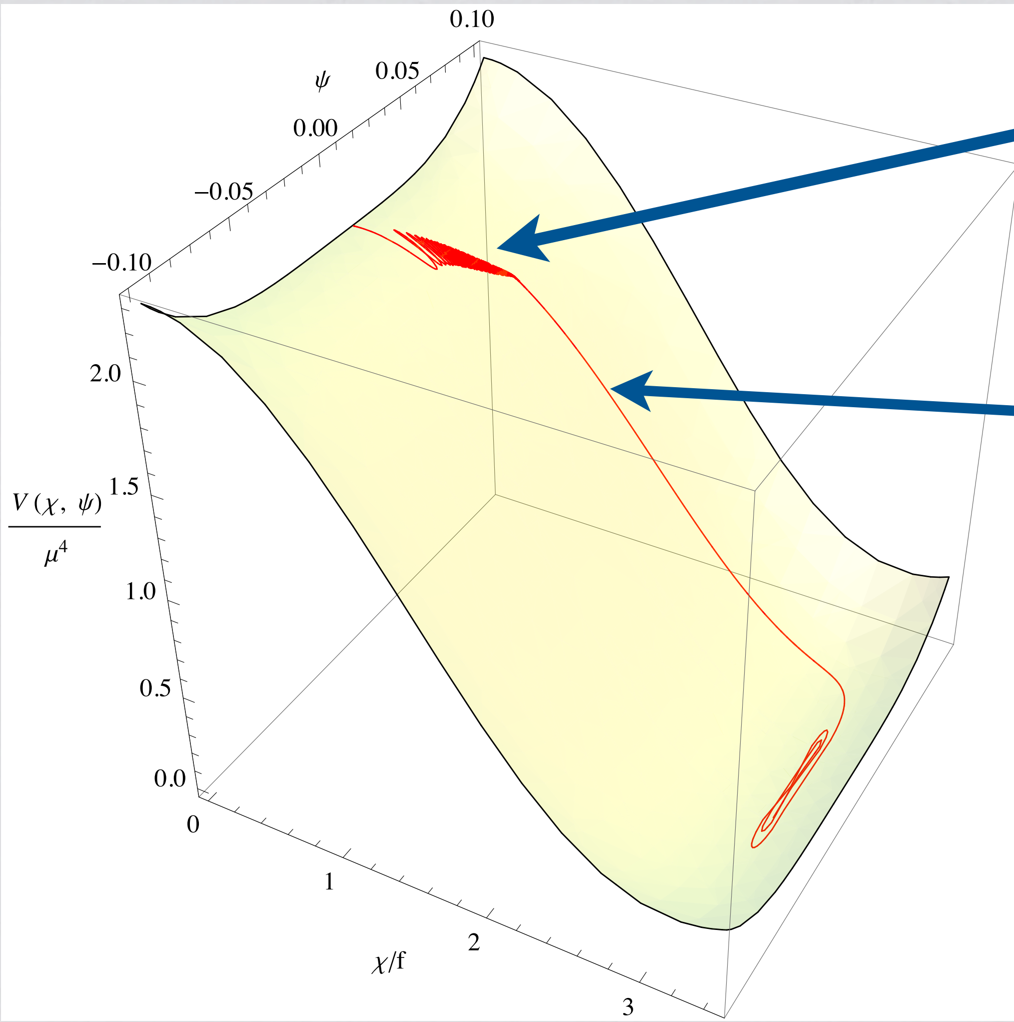
Chromo-Natural Drift



Fast `Larmor' oscillations

Magnetic drift

$$\text{``}B\text{''} = \frac{\lambda}{f} g\psi^2$$

No eta problem

- In this scenario, the shape of the potential doesn't matter.
- Slow roll inflation due to hierarchy between the "magnetic" field strength and height of potential.

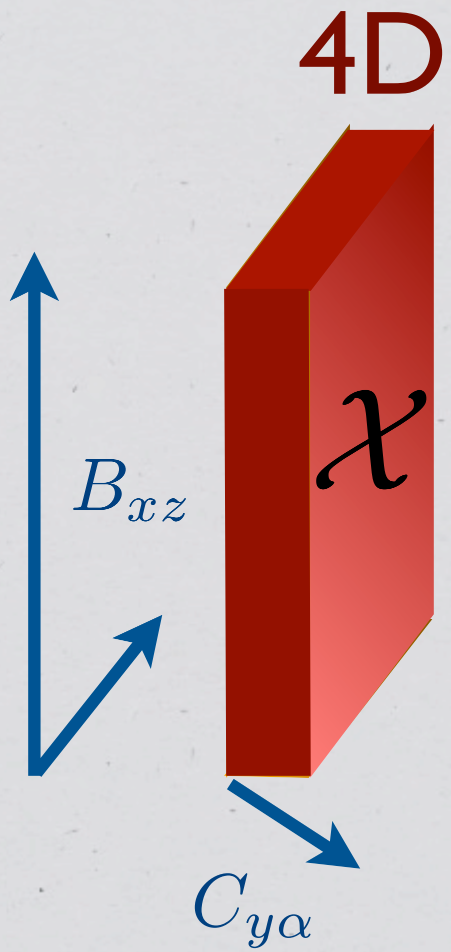
$$\frac{\lambda g}{f}\psi^2 \gg \sqrt{3}H$$

- Amounts to choosing $\lambda >> 1$
- Hierarchy is technically natural for this theory.

Possible concerns

- Confinement or gauge field dominating stress-energy?
  - ☑ OK if the gauge coupling g is small.
  - ☑ Provides graceful exit at the end of inflating era
- Catastrophic decay of gauge background?
  - ☐ Still calculating, but it appears gauge constraint projects dangerous tachyons out of the spectrum.

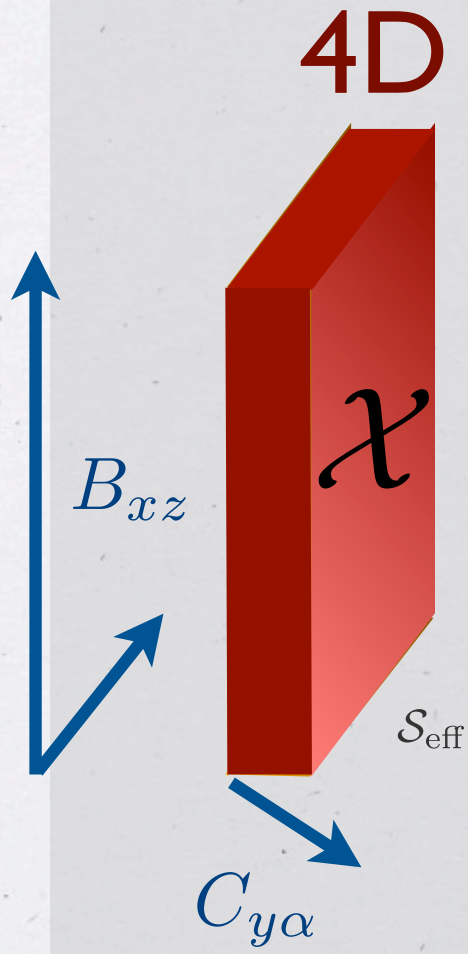
Another possibility: String theory!



$$\mathcal{L} \subset \partial_0 \mathcal{X} \varepsilon^{0xyz} B_{xy} C_{y\alpha}$$

"bulk" B and C fields can slow down the brane's motion!

D7-brane example



- D7 brane coupled to bulk fields in type IIB supergravity compactified to 4D

- Effective 4D action:

$$\mathcal{S}_{\text{eff}} = \int dtd^3x\sqrt{-g}\Big[\Big(-\frac{R}{2}-\frac{\gamma_b}{4}(\partial_\mu B_{\nu\rho})^2-\frac{\gamma_c}{2}(\partial_\mu C_{\nu\alpha})^2-\frac{\gamma_X}{2}(\partial X^\alpha)^2\Big)-\mathcal{V}(X)\Big]$$
$$-\lambda\int d^3xdt\,\epsilon^{ijk}B_{ij}C_{k\alpha}\partial_t X^\alpha$$

D7 Magnetic Drift

- Ansatz for gauge fields:

$$B_{12} = a^2 b,\ C_3 = ac$$

- Diagonalize eom in slow roll limit

$$[9\gamma_b\gamma_c\gamma_X H^2 + \lambda^2(\gamma_b b^2 + \gamma_c c^2)]\dot{X} = -\gamma_b\gamma_c H(3\mathcal{V}' + 5H\lambda bc)$$

$$\left[9\gamma_b\gamma_c\gamma_X H^2 + \lambda^2(\gamma_b b^2 + \gamma_c c^2)\right]\dot{c} = -\left[\gamma_b\lambda b\mathcal{V}' + c\Big(\lambda^2 H\big(\frac{2}{3}\gamma_c c^2 + \frac{7}{3}\gamma_b b^2\big) + 6\gamma_b\gamma_c\gamma_X H^3\Big)\right]$$

$$\left[9\gamma_b\gamma_c\gamma_X H^2 + \lambda^2(\gamma_b b^2 + \gamma_c c^2)\right]\dot{b} = -\left[\gamma_c\lambda c\mathcal{V}' + b\Big(\lambda^2 H\big(\frac{7}{3}\gamma_c c^2 + \frac{2}{3}\gamma_b b^2\big) + 6\gamma_b\gamma_c\gamma_X H^3\Big)\right]$$

- System will undergo magnetic drift for $\dfrac{H}{\lambda} \ll 1$
- B and C dynamics undamped - fast evolution to fixed point

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'magnetic friction' terms

- System will undergo magnetic drift for $\dfrac{H}{\lambda} \ll 1$
- B and C dynamics undamped - fast evolution to fixed point

D7 Magnetic Drift

- B and C dynamics undamped - fast evolution to the fixed point

$$b^* \approx \left(\frac{\mathcal{V}'}{3\lambda H} \sqrt{\frac{\gamma_c}{\gamma_b}} \right)^{1/2} , \quad c^* \approx \left(\frac{\mathcal{V}'}{3\lambda H} \sqrt{\frac{\gamma_b}{\gamma_c}} \right)^{1/2} .$$

- At this fixed point, brane undergoes slow magnetic drift:

$$\frac{\dot{X}}{H} \approx 2\sqrt{\gamma_b \gamma_c}\frac{H}{\lambda}$$

- Again, parametrically suppressed by the ratio $\frac{H}{\lambda} \ll 1$

Anisotropy? Yes, but small.

- Insert axially symmetric Bianchi type-I metric

$$ds^2 = -\,dt^2 + e^{2\alpha(t)}(e^{-4\sigma(t)}dx^2 + e^{2\sigma(t)}(dy^2 + dz^2))$$

- Find: $$\frac{\dot{\sigma}}{H} = -\frac{\gamma_b b^2}{(3 + 2(\gamma_c c^2 + \gamma_b b^2))} \approx \frac{\mathcal{V}'}{3\lambda H}\sqrt{\gamma_b \gamma_c}$$

- Anisotropy $\mathcal{O}(\epsilon)$

- **Note**: anisotropy absent in 'chromo-natural' case

Perturbations?

- Usual story

Need: $$\hat{m} \sim \frac{V''}{V} \ll 1$$

- Here: two-field composite "light" direction:

$\delta\psi, \delta\mathcal{X}$ combination that preserves $$\psi = \left(\frac{V'}{3g\lambda H}\right)^{1/3}$$

always exists in the 'magnetic drift' limit!

Preliminary details:

$$\partial_\tau^2 \chi + k^2 \chi - \frac{2 - m_\chi^2}{\tau^2}\chi = \frac{\hat{\lambda}}{H}\left[\frac{\partial_\tau \beta}{\tau} + 2\frac{\beta}{\tau^2}\right]$$

$$\chi = a\delta\mathcal{X}$$
$$\beta = a\delta\psi \text{ or } a\delta(B_{ij} + C_k)$$

$$\partial_\tau^2 \beta + k^2 \beta - \frac{2 - m_\beta^2}{\tau^2}\beta = -\frac{\hat{\lambda}}{H}\left[\frac{2\partial_\tau \chi}{\tau} + 2\frac{\chi}{\tau^2}\right]$$

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Look for k=0 power law, $\chi = C_\chi\tau^\alpha,\ \beta = C_\beta\tau^\alpha$

$$(\alpha(\alpha-1) - (2-m_\chi^2))C_\chi = \frac{\hat{\lambda}}{H}(\alpha+2)C_\beta$$

$$(\alpha(\alpha-1) - (2-m_\beta^2))C_\beta = -\frac{\hat{\lambda}}{H}(2\alpha+2)C_\chi$$

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$$(\alpha(\alpha - 1) - (2 - m_\chi^2))C_\chi = \frac{\hat{\lambda}}{H}(\alpha + 2)C_\beta$$

$$(\alpha(\alpha - 1) - (2 - m_\beta^2))C_\beta = -\frac{\hat{\lambda}}{H}(2\alpha + 2)C_\chi$$

Late time growing mode (α≈-1) guaranteed when $\frac{\hat{\lambda}}{H} \to \infty$

Summary

- ‘Magnetic drift’ physics → slow roll inflation
- Mediated by Chern-Simons interactions
- 4D Chromo-Natural model, many string theory candidates
- Perturbations definitely scale-invariant
  - Still working on amplitude calculation