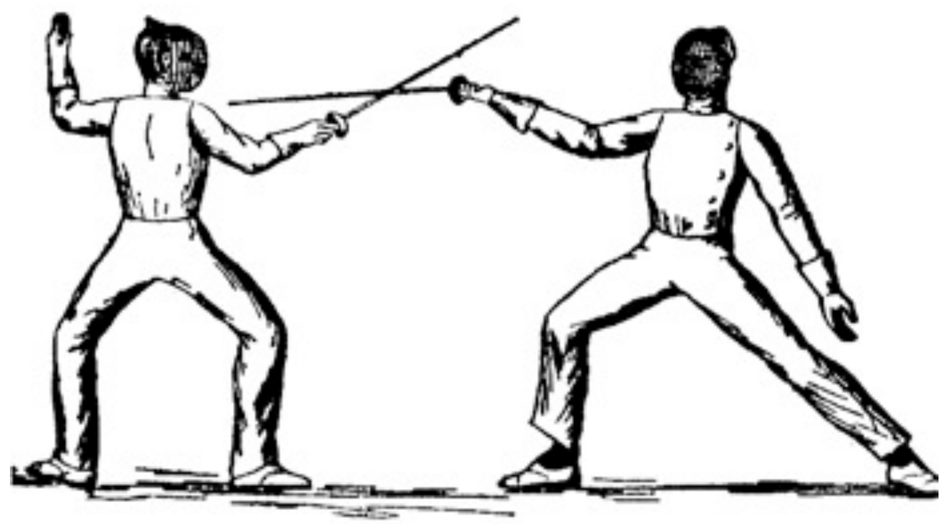
Higuchi VS Vainshtein



ArXiv: 1206.3852+1208. ????, soon!

Matteo Fasiello and Andrew J. Tolley

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Outline

1) the Higuchi bound

2) the Vainshtein mechanism

3) Higuchi vs Vainshtein

4) some relevant examples in the literature

5) dRGT setup

6) dS on dS

7) FRW on FRW

8) despair not !

9) conclusions and future work

The Higuchi bound is a condition that stems from requiring stability from the classical theory of linear Massive Gravity

(1)

$$\mathcal{L} = \mathcal{L}_{EH} + \mathcal{L}_m = \sum p^T \dot{q} - \left[\frac{1}{2} p^T \cdot P \cdot p + \frac{1}{2} q^T \cdot Q \cdot q + p^T \cdot \bar{P}Q \cdot q\right]$$

Roughly speaking: stability <==> Q , P positive definite (Higuchi + gradient instability)

(2)

Essential literature:

A. Higuchi
Nucl.Phys. B282 (1987) 397

S. Deser, A. Waldron
Phys.Lett. B508 (2001) 347-353
hep-th/0103255

L.Grisa, L.Sorbo
Phys.Lett. B686 (2010) 273-278
arXiv:0905.3391

Let's take a look

example: Fierz-Pauli

$$S = S_{EH} - \frac{m^2}{4}\int d^4x\sqrt{-\bar{g}^{(4)}}h_{\mu\nu}h_{\rho\sigma}\left[f^{\mu\rho}f^{\nu\sigma} - f^{\mu\nu}f^{\rho\sigma}\right]$$

where:

$$f^{\mu\nu} = \bar{g}^{\mu\nu}_{EH}$$

- usual tensor decomposition $T_{ij} = T^{Tt}_{ij} + 2\partial_{(i}T^{t}_{j)} + \frac{1}{2}\left(\delta_{ij} - \hat{\partial}_{ij}\right)T^t + \hat{\partial}_{ij}T^l$

We are looking at the scalar here, the helicity 0 mode

- use ADM formalism
- solve constraint equations, solve for p^t, h^t
- canonical transformation: $p^l \rightarrow p_0 + h^t\left(m^2 - 2H^2\right)/4H \;\; ; \;\; h^l \rightarrow q_0 + h^t/2$

$$I_0 = p_0 \dot{q}_0 - \Big[\frac{1}{2}\Big[\frac{3\nu^2 m^2}{12H^2}\Big]p_0^2 + \frac{1}{2}\Big[\frac{12H^2}{\nu^2 m^2}\Big] q_0 \left(-\nabla^2 + m^2 - \frac{9H^2}{4}\right) q_0\Big]$$

$$\nu^2 = m^2 - 2H^2$$

Immediately then, stability dictates:

$$\nu^2 > 0$$

in this setup, the Higuchi bound reads:

$$\boxed{m^2 > 2H^2}$$

Vainshtein radius

a quick derivation: $$R_{\mu\nu} + m^2 h_{\mu\nu} \sim T_{\mu\nu}$$

$$h_{\mu\nu} \sim 1 \qquad\qquad R \sim m^2$$

$$R \sim \nabla^2 \phi \; ; \qquad \phi \sim \frac{GM}{r} \Rightarrow R \sim \frac{GM}{r^3} \sim m^2$$

therefore

$$r < r_V \longleftarrow \boxed{r_V = \left(\frac{M}{M_P^2 m^2}\right)^{1/3}} \longrightarrow r > r_V$$

Vainshtein screening mechanism at work here, remember previous talks

C. Deffayet, G. Dvali, G. Gabadadze, A. Vainshtein hep-th/0106001
Phys.Rev. D65 (2002) 044026

G. Chkareuli, D. Pirtskhalava
airXiv 11.05.1783

Vainshtein Mechanism

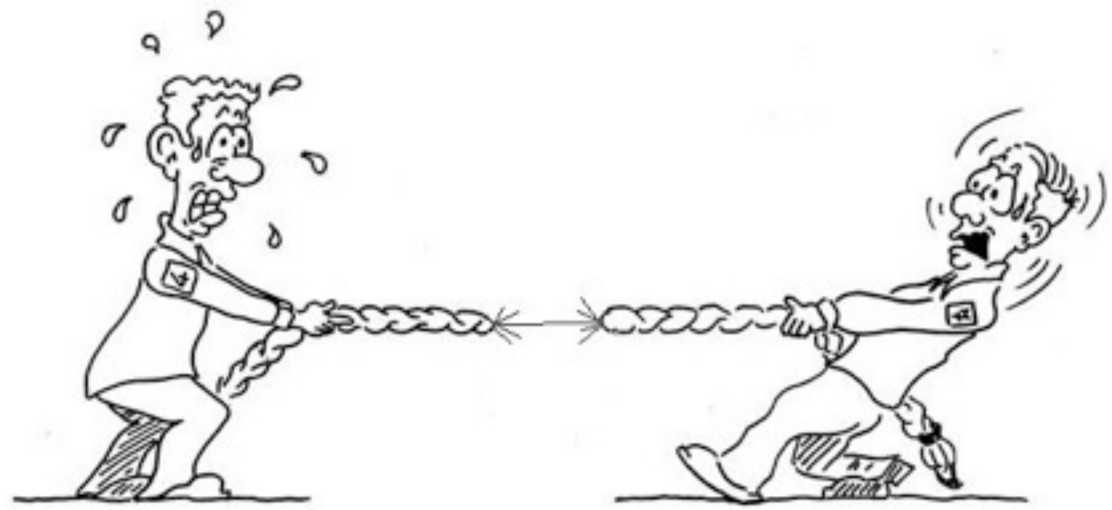
remember from above: inside the Vainshtein radius lies the region where one recovers GR,

schematically, when presented, as we will be soon, with :

$$3H^2 = \Lambda + 3m^2 \times \Theta(1)$$

one must require

$$m^2 < H^2$$



want our theory to be stable

GR works all around us

$$m^2 > 2H^2 \qquad \overset{*}{-} \qquad m^2 < H^2$$

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Nucl.Phys. B282 (1987) 397

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A. I. Vainshtein
Phys.Lett. B39 (1972) 393-394

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Clearly, there's a problem...

But note that, in deriving the Higuchi bound before, a number of assumptions have been implicitly made:

*** Shall we add matter content?

*** Shall we use a different reference metric "f"?

*** F-P theory of MG has ghosts. How about a ghost free theory?

*** Something else?

Let's add matter:

$$S = S_{EH} + S_{m^2} - \int d^4x \sqrt{-g^{(4)}} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + V(\Phi) \right]$$

keeping the assumption:

$$f^{\mu\nu} = \bar{g}^{\mu\nu}$$

The Higuchi bound now reads

$$m^2 > 2(H^2 + \dot{H})$$

L.Grisa, L.Sorbo
Phys.Lett. B686 (2010) 273-278
arXiv:0905.3391

but remember, Fierz-Pauli theory has ghosts  !

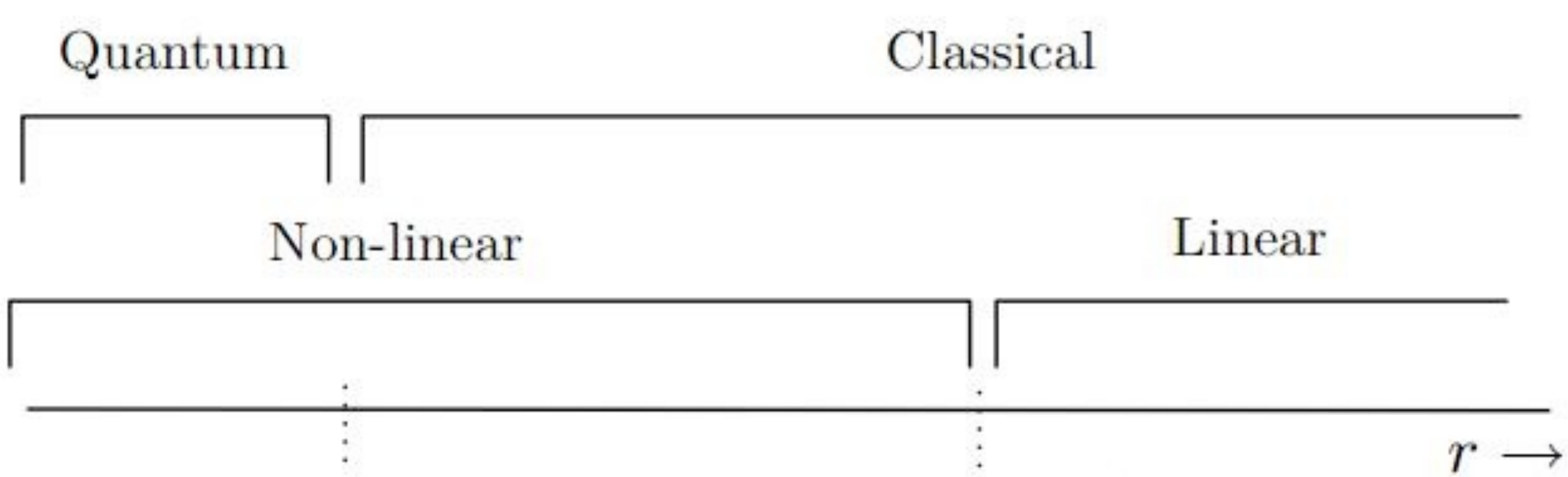
dRGT: Ghost-free m.g. theory at fully non-linear level

De Rham, Gabadadze, Tolley
Hassan, Rosen

* No Boulware-Deser Ghost, at all orders

* Screening mechanism in the non linear regime that restores continuity with G.R. as m approaches 0

* High enough cutoff so that the theory different regimes can be described



$$S = S_{EH} + 2m^2 \int d^4x \sqrt{-g} \left[\varepsilon_2(\delta - \sqrt{g^{-1}f}) + \alpha_3\, \varepsilon_3(\delta - \sqrt{g^{-1}f}) + \alpha_4\, \varepsilon_4(\delta - \sqrt{g^{-1}f}) \right]$$

Our set up

* dRGT theory of massive gravity

$$S_{m^2} = 2m^2 \int d^4x\sqrt{-g}\Big[\varepsilon_2(\delta - \sqrt{g^{-1}f}) + \alpha_3\,\varepsilon_3(..) + \alpha_4\,\varepsilon_4(..)\Big]$$

with

$$\varepsilon_2(X) = \frac{1}{2}\left(Tr^2[X] - Tr[X^2]\right);$$

$$\varepsilon_3(X) = \frac{1}{6}\left(Tr^3[X] - 3Tr[X^2]Tr[X] + 2Tr[X^3]\right)$$

$$\varepsilon_4(X) = \frac{1}{24}\left(Tr^4[X] - 6Tr[X^2]Tr^2[X] + 3Tr^2[X^2] + 8Tr[X^3]Tr[X] - 6Tr[X^4]\right)$$

** The reference metric "f" and "g_0" need not be the same, parametrize this as:

$$f_{\mu\nu} = (1+z)\bar{g}_{\mu\nu}{}^{*}$$

* *in dS*

dS on dS

* *alpha's set to 0 for simplicity*

Friedman equation:

$$3H^2 = m^2(3z - 3z^2) + \Lambda$$

therefore:

$$m^2(z - z^2) \lesssim H^2$$

modified Higuchi bound:

$$m^2(1 - z - 2z^2)(m^2(1 - z - 2z^2) - 2H^2) > 0$$

overall then:

$$\frac{1 - z - 2z^2}{3z - 3z^2} \gg 1$$

Higuchi bound:

$$m^2(1-z-2z^2)(m^2(1-z-2z^2)-2H^2)>0$$

in other words, the Higuchi bound has the generic form

$$\tilde{m}^2(\tilde{m}^2-2H^2)>0$$

$\tilde{m}$ *is the dressed mass, we ask* $\tilde{m}^2>0$ *to avoid instabilities in the vector sector.*

Two branches of solutions:

$$0<H<\frac{3}{2}H_0 \quad ; \qquad m^2>\frac{2HH_0^2}{3H_0-2H} \; .$$

includes the H₀ =H branch

$$H>\frac{3}{2}H_0\,; \qquad m^2<-\frac{2HH_0^2}{2H-3H_0} \; .$$

* $1+z=H/H_0$

new branch

apparently, for H>>H₀, $|m^2|/H_0^2>1$

this is a much weaker Higuchi bound, but Vainshtein will require the opposite inequality to hold, a.k.a. : back to square 1.

Let's switch on α_3 , α_4

e.o.m. :

$$H^2 = \frac{1}{3}\Lambda + m^2(z - z^2) - \alpha_3 m^2(z^2 - \frac{z^3}{3}) + \frac{1}{3}\alpha_4 m^2 z^3$$

Higuchi bound:

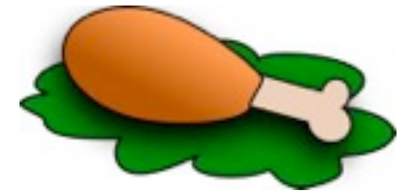
$$m^2(1+z)(1 - z(2 - \alpha_3(-2+z) - \alpha_4 z)) \geq 2H^2$$

The combined requirement now reads:

$$\frac{1 - z - 2z^2 - 2\alpha_3 z - \alpha_3 z^2 + \alpha_4 z^2 + \alpha_3 z^3 + \alpha_4 z^3}{3z - 3z^2 - 3\alpha_3 z^2 + \alpha_3 z^3 + \alpha_4 z^3} \gg 1$$

This inequality can in principle be satisfied, but for specifically tuned values of the parameters, which is somewhat unnatural.

Add matter:



$$L = L_{EH} + L_{dRGT} + \int d^3x \sqrt{-{}^{(4)}g} \left(-\frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + V(\Phi) \right)$$

Background eq:

$$H^2 = m^2(z - z^2) + \frac{\bar{\pi}^2}{12} + \frac{V_0}{6}$$

$$\dot{H} = -\frac{\bar{\pi}^2}{4} - \frac{m^2}{2} \left(1 - z - 2z^2 - M + 2Mz\right)$$

$$\dot{\bar{\pi}} + 3H\bar{\pi} + V_1 = 0 \; ; \quad V_1 = \frac{dV(\phi)}{d\phi}$$

$$\dot{z} = -\frac{H}{1 - 2z} \left(1 - z - 2z^2 - M + 2Mz\right)$$

$$^{*} f_{\mu\nu} = \mathrm{diag}\left[-\mathrm{M}^2(\mathrm{t}), (1 + \mathrm{z}(\mathrm{t}))^2\right]$$

$\dot{H}$ *drops out of the Higuchi inequality !*

The bound is independent from the equation of state for matter

$$\tilde{m}^2(H) = m^2 \frac{H}{H_0}\left((3+3\alpha_3+\alpha_4) - 2(1+2\alpha_3+\alpha_4)\frac{H}{H_0} + (\alpha_3+\alpha_4)\frac{H^2}{H_0^2}\right) \geq 2H^2.$$

the problem is still there !

$$\alpha_3 = -1 = -\alpha_4$$

One might hope that getting playful with the alpha's could pay off:

it doesn't .Time evolution does not help either.

$$\frac{\mathrm{poly}_1^{(k)}(z)}{\mathrm{poly}_2^{(k)}(z)} \gg 1$$

structure also makes it hard.

Message

Higuchi vs Vainshstein tension cannot be relaxed in this, quite generic, setup*,

- not by using two different FRW metrics
- not by adding matter

* *partially massless*

In this setup there is no regime which is simultaneously observationally acceptable and ghost-free.

What to do now?

<u>Fully bi-metric theories</u> ✓

done !

In bigravity theories the H-V tension is relaxed for a condition as simple as H<<H0.

To appear very soon !

$$m^2_{\rm dressed}(H)\left(H^2+\frac{H_0^2M_P^2}{M_f^2}\right)\geq 2H^4$$

new !

<u>Inhomogeneities</u> in the ϕ's ?

work in progress...

Reasons to be hopeful: see D'Amico et al.,"massive cosmologies".