Structure formation in a nonlocally modified gravity model

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What's the problem I consider:

- Friedmann eqn
$$3H^2(t) = 8\pi G\rho(t)$$
Data says L.H.S $H(t)$ approaching const. but R.H.S $\rho(t) = \frac{\rho_{matter}}{a^3(t)}$ falling off
- To fix this problem,
  - Solution 1: add more energy on the r.h.s. $\rightarrow$ dark energy
  - Solution 2: modifiy the l.h.s. $\rightarrow$ modified gravity
- These can be tested by looking at the growth of structure

Nonlocally modified gravity model using $\Box^{-1} R$

- Motivation: "The only stable, local, pure-metric modifications of gravity in $D = 4$ dimensions are $f(R)$ models," Woodard 2006
  allow nonlocal Lagrangian: might derive from quantum corrections
  Parker and Toms 1985, Banks 1988, Hamber and Williams 2005
- Model: S. Deser and R. P. Woodard, PRL **99** (2007) 111301, arXiv:0706.2151.
  Modify the Einstein-Hilbert Lagrangian: $\mathcal{L}_g = \frac{1}{16\pi G}\sqrt{-g}R\left[1 + f(\frac{1}{\Box}R)\right]$
- Main virtues
  1. $f\left(\frac{1}{\Box}R\right)$ is dimensionless $\rightarrow$ No new mass parameter.
  2. $\frac{1}{\Box}R$ grows slowly $\rightarrow$ delays the onset of acceleration.
- $\frac{1}{\Box}\overline{R} = -\int_{t_i}^{t} \frac{dt'}{a^3(t')} \int_{t_i'}^{t'} dt'' a^3(t'')\left[6\dot{H}(t'') + 12H^2(t'')\right]$
  $\sim \ln(\frac{t}{t_i})$ for matter-dom., $a(t) \sim t^{2/3}$

The modified field equation

- The nonlocally modified Lagrangian: $\mathcal{L}_g = \frac{1}{16\pi G}\sqrt{-g}R\Big[1 + f(\frac{1}{\Box}R)\Big]$
- Vary the action w.r.t $g_{\mu\nu}$ to get the modified field equation:
$G_{\mu\nu} + \Delta G_{\mu\nu} = 8\pi G T_{\mu\nu}$
$\Delta G_{\mu\nu} = \Big[G_{\mu\nu} + g_{\mu\nu}\Box - D_\mu D_\nu\Big]\Big\{f\Big(\frac{1}{\Box}R\Big) + \frac{1}{\Box}\Big[Rf'\Big(\frac{1}{\Box}R\Big)\Big]\Big\} + \Big[\delta^{(\rho}_{\mu}\delta^{\sigma)}_{\nu} - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma}\Big]\partial_\rho\Big(\frac{1}{\Box}R\Big)\partial_\sigma\Big(\frac{1}{\Box}\Big[Rf'\Big(\frac{1}{\Box}R\Big)\Big]\Big)$.
- The zeroth order eqn for the background FRW metric:
$3H^2 + \Delta G_{00} = 8\pi G\rho,$
$-2\dot{H} - 3H^2 + \frac{1}{3a^2}\delta^{ij}\Delta G_{ij} = 8\pi G P$
- $\Delta G_{00} = \Big[3H^2 + 3H\partial_t\Big]\Big\{f\Big(\frac{1}{\overline{\Box}}\overline{R}\Big) + \frac{1}{\overline{\Box}}\Big[\overline{R}f'\Big(\frac{1}{\overline{\Box}}\overline{R}\Big)\Big]\Big\} + \frac{1}{2}\partial_t\Big(\frac{1}{\overline{\Box}}\overline{R}\Big)\partial_t\Big(\frac{1}{\overline{\Box}}\Big[\overline{R}f'\Big(\frac{1}{\overline{\Box}}\overline{R}\Big)\Big]\Big)$
$\Delta G_{ij} = -a^2\delta_{ij}\Big[2\dot{H} + 3H^2 + 2H\partial_t + \partial_t^2\Big]\Big\{f\Big(\frac{1}{\overline{\Box}}\overline{R}\Big) + \frac{1}{\overline{\Box}}\Big[\overline{R}f'\Big(\frac{1}{\overline{\Box}}\overline{R}\Big)\Big]\Big\} + a^2\delta_{ij} \times \frac{1}{2}\partial_t\Big(\frac{1}{\overline{\Box}}\overline{R}\Big)\partial_t\Big(\frac{1}{\overline{\Box}}\Big[\overline{R}f'\Big(\frac{1}{\overline{\Box}}\overline{R}\Big)\Big]\Big)$

The zeroth order eqn and the nonlocal distortion function f

- The Zeroth order equation gives late time acceleration:
  The free function f is constructed to give $a(t)$ of ΛCDM
  C. Deffayet and R. P. Woodard, JCAP **08** (2009) 023, arXiv:0904.0961.
  For ΛCDM, with the choice of $\{\Omega_\Lambda, \Omega_m, \Omega_r\} = \{0.72, 0.28, 8.5 \times 10^{-5}\}$,

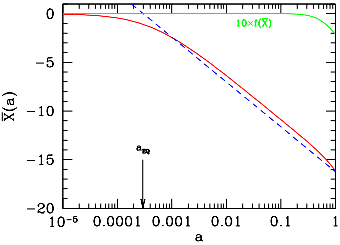

Figure: The evolution of $\overline{X} \equiv \square^{-1}\overline{R}$. Dashed line shows the analytic solution in the matter era: $\overline{X} = -2\ln(a/a_{\rm EQ})$. The flat (green) curve shows the function $f(\bar{X})$ chosen to fit the ΛCDM expansion history. Its amplitude and shape are chosen so that it has negligible impact on the expansion until relatively recently.

Perturbation Equations

- To see the growth of structure, perturb the metric around the FRW, with the gauge choice of Dodelson, S. Dodelson, *Modern Cosmology*: $g_{00}(t,\vec{x}) = -1 - 2\Psi(t)e^{i\vec{k}\cdot\vec{x}}\,, \quad g_{0i}(t,\vec{x}) = 0\,, \quad g_{00}(t,\vec{x}) = \delta_{ij}a^2(t)\Big[1 + 2\Phi(t)e^{i\vec{k}\cdot\vec{x}}\Big]$
- The modified Einstein field equations:

$$
\begin{aligned}
G_{00} + \Delta G_{00} &= 8\pi G T_{00} \\
\overline{G}_{00} + \Delta\overline{G}_{00} + \delta\Big(G_{00} + \Delta G_{00}\Big) &= 8\pi G\Big(\overline{T}_{00} + \delta T_{00}\Big) \\
G_{ij} + \Delta G_{ij} &= 8\pi G T_{ij} \\
\overline{G}_{ij} + \Delta\overline{G}_{ij} + \delta\Big(G_{ij} + \Delta G_{ij}\Big) &= 8\pi G\Big(\overline{T}_{ij} + \delta T_{ij}\Big)
\end{aligned}
$$

- The FRW background equation (showed in the previous slide) $\overline{G}_{00} + \Delta\overline{G}_{00} = 8\pi G\overline{T}_{00}, \quad \overline{G}_{ij} + \Delta\overline{G}_{ij} = 8\pi G\overline{T}_{ij}$
- Perturbation equation (will show in the next slide) $\delta\Big(G_{00} + \Delta G_{00}\Big) = 8\pi G\delta T_{00}, \quad \delta\Big(G_{ij} + \Delta G_{ij}\Big) = 8\pi G\delta T_{ij}$

Perturbation Equations (continued)

- Poisson equation

$$k^2\Phi + k^2\Phi\left\{f(\overline{X}) + \frac{1}{\overline{\Box}}\left[\overline{R}f'\left(\overline{X}\right)\right]\right\} + \frac{k^2}{2}\left\{f'(\overline{X})\frac{1}{\overline{\Box}}\delta R + \frac{1}{\overline{\Box}}\left[f'\left(\overline{X}\right)\delta R + \overline{R}f''\left(\overline{X}\right)\frac{1}{\overline{\Box}}\delta R\right]\right\}$$
$$+\frac{1}{4}a^2\partial_t(\overline{X})\partial_t\left(\frac{1}{\overline{\Box}}\left[f'\left(\overline{X}\right)\delta R + \overline{R}f''\left(\overline{X}\right)\frac{1}{\overline{\Box}}\delta R\right]\right) + \frac{1}{4}a^2\partial_t(\frac{1}{\overline{\Box}}\delta R)\partial_t\left(\frac{1}{\overline{\Box}}\left[\overline{R}f'\left(\overline{X}\right)\right]\right)$$
$$= 4\pi Ga^2\bar{\rho}\delta$$

- Slip equation

$$(\Phi + \Psi) = 8\pi G\delta T_B - (\Phi + \Psi)\left\{f(\overline{X}) + \frac{1}{\overline{\Box}}\left[\overline{R}f'\left(\overline{X}\right)\right]\right\}$$
$$-f'(\overline{X})\frac{1}{\overline{\Box}}\delta R - \frac{1}{\overline{\Box}}\left[f'\left(\overline{X}\right)\delta R + \overline{R}f''\left(\overline{X}\right)\frac{1}{\overline{\Box}}\delta R\right]$$

Solution of the Poisson equation

- Note the new terms due to the non-local interactions are small: solve the equation perturbatively by inserting the standard solution for Φ and Ψ into the new terms
- The modified Poisson eqn: $k^2\Phi + k^2 E[\Phi] = 4\pi G a^2 \bar{\rho}\delta$ where $E[\Phi]$: the new terms
- Define the effective Newton's constant G_{eff} as $k^2\Phi = 4\pi G_{eff} a^2 \bar{\rho}\delta = G_{eff}\frac{k^2\Phi + E[\Phi]}{G}$.
- The fractional change G_{eff}/G is $\frac{G_{eff}}{G} = \frac{k^2\Phi}{k^2\Phi + E[\Phi]} = \frac{1}{1+\frac{E[\Phi]}{k^2\Phi}}$

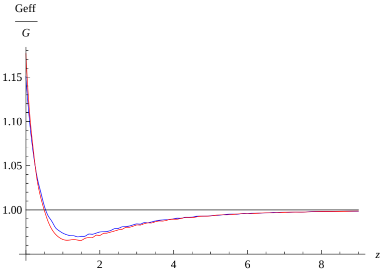

Figure: The fractional change to Newton's constant, $\frac{G_{\mathrm{eff}}}{G}$ as a function of redshift z. The two curves, which depict the evolution for $k = 0.03, 0.3$h Mpc^{-1}, show that the modification is virtually scale-independent, at least in the linear regime.

Solution for the slip equation

- To see the deviation from isotropy $(\Phi + \Psi) = 0$, set $\Psi = -\Phi$ on the r.h.s and again use the standard solution for Φ and Ψ.
- Fractional deviation, $(\Phi + \Psi)/\Phi$:

$$(\Phi + \Psi)/\Phi = \left\{ -f'(\overline{X})\delta X - \frac{1}{\overline{\Box}} \left[f'\left(\overline{X}\right)\delta R + \overline{R} f''\left(\overline{X}\right)\delta X - \delta\Box \frac{1}{\overline{\Box}} \left[\overline{R} f'\left(\overline{X}\right) \right] \right] \right\} / \Phi$$

- The amount of deviation for different k's turns out almost the same.

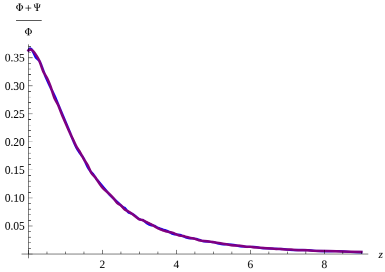

Figure: Gravitational slip as a function of redshift in the nonlocal model. The two curves, barely distinguishable, are for $k = 0.03$ h Mpc^{-1} and $k = 0.3$ h Mpc^{-1} .

Summary

- Studied a nonlocally modified gravity model proposed by Deser and Woodard which gives an explanation for current cosmic acceleration.
- Showed that this model predicts a pattern of growth that differs from standard general relativity (+dark energy) at the 10-30% level.
- These differences will be easily probed by the next generation of galaxy surveys, so the model should be tested shortly.