

Essays on Urban Agglomeration

Jeffrey Brinkman
Carnegie Mellon University

August 29, 2011

Abstract

This thesis examines several distinct aspects of agglomeration externalities within an urban economy. In particular, the work focuses on location decisions of agents who face trade-offs between advantages in dense locations versus cheaper land prices available in suburban and exurban regions of the city. Two general equilibrium models of location are presented which study different aspects of location choice in urban areas.

In chapter 1, we develop a new dynamic general equilibrium model to explain firm entry, exit, and relocation decisions in an urban economy with multiple locations. We characterize the stationary distribution of firms that arises in equilibrium. The parameters of the model can be estimated using a nested fixed point algorithm by matching the observed distribution of firms by location and the one implied by our model. We implement the estimator using unique data collected by Dun and Bradstreet for the Pittsburgh metropolitan area. Firms located in the central business district are older and larger than firms located outside the urban core. They use more land and labor in the production process. However, they face higher rental rates for office space which implies that they operate with a higher employee per land ratio. Our estimates imply that agglomeration externalities increase the productivity of firms by one to two percent. Economic policies that subsidize firm relocations can potentially have large effects on economic growth and firm concentration in central business districts.

In Chapter 2, I develop and estimate a general equilibrium model of business and residential location in the presence of agglomeration externalities and commuting costs. The model is based on the theory introduced by Lucas and Rossi-Hansberg (2002), but adds a congestion cost in addition to a distance cost of commuting. This specification allows for the investigation of the effect of different transportation technologies (i.e. transit or automobile infrastructure) on the spatial structure of cities. In addition, other modifications are made in order to make empirical analysis tractable. I introduce data on commercial

and residential densities, commuting times, and wages paid, to illustrate the structure of cities and highlight the trade offs faced by businesses and individuals in location decisions. The model is estimated using a method of moments procedure, and the estimates are used to illustrate the quantitative aspects of equilibrium, including the importance of congestion in commuting costs. Policy experiments show that decreasing congestion costs relative to distance costs (a policy akin to increasing transit provision) increases the relative concentration of employment in the center city and increases residential density in inner ring suburbs.

Acknowledgments

I wish to thank ...

Contents

1	Introduction	6
2	Agglomeration Externalities and the Dynamics of Firm Location Choices within an Urban Economy	10
2.1	Introduction	10
2.2	Data	14
2.3	A Dynamic Model of Firm Location within an Urban Area	19
2.3.1	Technologies and Markets	19
2.3.2	Equilibrium	23
2.4	Estimation	29
2.5	Some Preliminary Estimation Results	31
2.6	Conclusions	37
2.A	Appendix: Firm Location Choices in a Sample of Large U.S. Cities	38
2.B	Appendix: Analytical Properties of Equilibrium	39
3	Transportation Technologies, Agglomeration, and the Structure of Cities	45
3.1	Introduction	45

3.2	The Structure of Cities	48
3.3	The Model	51
3.4	Computation of Equilibrium	59
3.5	Estimation	63
3.6	Quantitative Results	67
3.7	Policy Considerations	69
3.8	Conclusions	73
	Bibliography	75

List of Figures

2.1	Distribution of Employment in Allegheny County	16
2.2	Graphical Representation of Equilibrium	27
2.3	Stationary Distributions and Decision Rules	34
3.1	Residential density, employment density, and wages paid as a function of radius from the city center for selected cities	50
3.2	Spatial distribution of employment, population, wages paid, and commuting times for Columbus, Ohio	52
3.3	Example of wage path construction for given production externality	62
3.4	Marginal commuting cost as a function of radius from the city center	68
3.5	Density (per square mile) and commuting costs (dollars per year by residential location) as a function of radius from the city center: theoretical model and data)	70
3.6	Change in employment density, residential density, and commuting costs relative to baseline for different transportation technologies	73

List of Tables

2.1	Employment and facility size by industry in 2008	15
2.2	Sorting of Firms by Location and Industry in 2008	18
2.3	Parameter Estimates	32
2.4	Rental Price Ratios and Relocation	33
2.5	Age-Employment distributions of establishments by location, as a percentage of total establishments in the entire county (computational moments in parenthesis)	35
2.6	Age-Facility Size establishment distributions by location as percentage of total establishments in the county, (computational moments in parenthesis	36
2.7	Concentration of employment in dense business districts	39
3.1	Employment and commuting mode characteristics of selected cities	48
3.2	Estimation results, n=322	68
3.3	Fit of the model: Computational densities and average commuting costs by location, (equivalents from data in parenthesis)	70
3.4	Changes in employment and population densities for different transportation technologies (Total employment held constant)	72

Chapter 1

Introduction

Economic activity is not uniformly distributed on any geographic scale. We observe firms clustered in global regions, metropolitan areas, and in specific neighborhoods or districts. Clear evidence suggests that a significant part of this clustering results from production externalities, also known as agglomeration externalities or production spillovers. In the most general definition, these externalities are simply the idea that firms gain some advantage in production by being located in close proximity to other firms. However, the specific details of these production advantages are the source of a large literature dedicated to the study of the magnitude, scope, and nature of agglomeration externalities. Some of this literature is reviewed in the following chapters.

The clustering of economic activity is particularly observable on the urban level. Both residential and commercial densities vary by orders of magnitude within even moderately sized metropolitan areas, with high density business districts proving particularly salient. Land rents also vary enormously within cities. These features suggest that workers and firms face trade-offs in their location decisions within cities - trading some production or consumption advantage in dense neighborhoods with the cheaper rents and more space available on the outskirts of the city. Given that natural locational advantages are less prevalent and labor is more mobile within urban areas compared to across metropolitan regions, consumption and production advantages become particularly relevant.

This thesis focuses on several distinct features and consequences of agglomeration externalities within urban areas. Chapter 2 examines the importance of dynamic decisions on the location choices of businesses within cities. Specifically, a theory is introduced and explored to explain the entry and exit of firms into a city and also the location decisions of firms over their lifecycle. Data analysis shows that businesses in dense business districts tend to be older and larger. In addition, there is evidence that businesses in dense business districts are more productive despite using less land or office space per employee. These characteristics suggest that over the lifecycle of a firm, the relative advantage of cheap rents versus production externalities change, with larger more productive firms being better able to take advantage of dense business districts than smaller less productive firms.

With these basic facts in mind, we develop a dynamic two-location model of firm location with agglomeration externalities. The two locations could be thought of as a dense business district and suburban business district, although location characteristics are determined endogenously in equilibrium. We also develop a nested fixed point algorithm to solve and characterize the stationary equilibrium of the model. This algorithm can then be used to estimate the model. Data for estimation comes from establishment level data from Dun and Bradstreet for the Pittsburgh Metropolitan area. The model is able to produce locational characteristics and sorting patterns similar to what is observed in the real economy. Because of the presence of agglomeration externalities, and the fact that relocation is not costless, our estimates suggest that there are potentially significant gains by subsidizing relocation costs or through other policies which tax or subsidize the two locations differently.

Chapter 3 considers a different aspect of urban location related to production externalities. While agglomeration externalities are an important driver of firm location choices, perhaps the primary source of clustering in residential location choices is transportation costs. This is not to say that other costs or benefits such as taxes and amenities, do not drive residential location choices, - they do, and probably on a larger magnitude than transportation costs - but the choice along the dimension of proximity to jobs versus access to cheap land, is presumably primarily driven by transportation costs. These two aspects of residential and commercial location preferences are at odds with one another, not just in

the competition over scarce land, but also in the fact that clustering of businesses inevitably leads to higher congestion costs in commuting.

This tension suggests a very interesting policy dilemma. While the existence of agglomeration externalities would suggest the implementation of subsidies for commercial density, this may be counterproductive in a general equilibrium analysis given that congestion costs represent a negative externality related to commercial density. Conversely, taxing congestion may have adverse effects on productivity. Instead, subsidies for transportation systems with reduced congestion costs may be a better solution.

In order to understand this question, I develop a general equilibrium model of commercial and residential location within an urban area. The model is based on the work by Lucas and Rossi-Hansberg (2002). The model includes agglomeration externalities and transportation cost, and includes the addition of a congestion cost to represent the congestion externalities imposed by commuters on one another. In addition, the transportation cost specification allows for the analysis of different types of transportation technologies, for instance transit technologies, which have low congestion costs, versus automobile infrastructure which has high congestion costs.

The model is solved computationally by using a shooting algorithm nested inside a fixed point algorithm. This solution algorithm enables estimation of the parameter vector by matching computational moments to moments observed in the data. The model is estimated using data from the Census Transportation Planning package, allowing for the analysis of commuting patterns. The estimated model captures important features of the real urban economy. First, business clustering is much more prevalent than residential cluster. In other words, very dense business districts tend to dissipate quickly across space suggesting that production externalities attenuate very quickly. Congestion costs on the other hand dissipate more slowly leading to less steep residential density gradients compared to commercial densities.

Overall, this thesis examines the effect of agglomeration externalities on the spatial allocation of economic activity within an urban area. First, the dynamics of firm location are

explored, including entry, exit and relocation decisions in the presence of agglomeration externalities. Then, I examine the interaction of firms and workers location choices in the face of competing externalities from agglomeration and congestion. Overall, the analysis suggests that agglomeration externalities, and policies to exploit or correct these externalities can have important consequences in welfare and the overall economic health of cities.

Chapter 2

Agglomeration Externalities and the Dynamics of Firm Location Choices within an Urban Economy¹

2.1 Introduction

Cities play an important role in the economy since economic proximity makes for more efficient production and trade. These efficiency gains typically arise because of agglomeration externalities.² Firms that operate in locations with high externalities have a competitive advantage over firms that are located in less efficient locations. Since firms will bid for the right to locate in areas with high agglomeration externalities, these locations have higher rental prices for land than locations that are less efficient.³ As a consequence, firms with

¹This chapter is joint work with Daniele Coen-Pirani (University of Pittsburgh) and Holger Sieg (University of Pennsylvania)

²The idea of geographic returns to scale was first introduced by von Thünen (1825). Marshall (1890) suggested that agglomeration effects may exist within industries. Firms may benefit from lower transaction costs or sharing of a common labor pool. Alternatively, efficiencies may arise due to positive diversity externalities and synergies between different industries (Jacobs, 1969).

³Krugman (1991) provides theoretical foundations for a two-location model of agglomeration. Ellison and Glaeser (1997) argued that agglomeration externalities are important to understand geographic concentration of manufacturing in the U.S. The literature of agglomeration theory is reviewed in Fujita and Thisse

different productivity levels will sort in equilibrium with high productivity firms locating in areas with high agglomeration effects and high rental prices for land. Low productivity firms are forced to exit the economy or operate in cheaper locations.

As the productivity of a firm changes over time, a firm's demand for land and labor changes as well. Moreover, productivity shocks create incentives to relocate within a city to exploit a better match with the agglomeration externalities. A firm that may have initially located outside a central business district may find it in its interest to move to a more densely populated central business district in order to grow and capture the full benefits of a persistent positive productivity shock. Similarly, a firm that has experienced a persistent negative shock must downsize and move to the urban fringe where land and labor is cheaper than in the central business district.⁴ The first objective of this paper is then to develop a new dynamic general equilibrium model of firm location choice that can explain the sorting of firms by productivity as well as entry, exit, and relocation decisions of firms in an urban economy with multiple locations.

We consider a model with two distinct locations which can be interpreted as inside and outside of a central business district (CBD). In equilibrium these locations differ in the magnitude of their agglomeration externalities which increase with employment density.⁵ Firms are heterogeneous in their productivities. We model firm dynamics and industry

(2002) and Duranton and Puga (2004). Anas and Kim (1996) and Lucas and Rossi-Hansberg (2002) have developed equilibrium models of mono- and poly-centric urban land use with endogenous congestion and job agglomeration. Rossi-Hansberg (2004) studies optimal land use policies in a similar framework.

⁴There is some evidence that shows that agglomeration effects are important to understand firm dynamics. Henderson, Kunkoro, and Turner (1995) show that agglomeration effects for mature industries are related to Marshall scale economies, while newer industries benefit from diversity akin to Jacobs economies. This work is important because it points to agglomeration as part of a dynamic process. Other research has continued to study the relevance of agglomeration in firm life-cycle dynamics. Duranton and Puga (2001) study the effect of agglomeration externalities in innovation and the development of production processes, while Dumais, Ellison, and Glaeser (2002) examine the effect of firm dynamics (entry, exit, expansion, and contraction) on the concentration of economic activity.

⁵Deckle and Eaton (1999) find that geographic scale of agglomeration is mostly at the national level, while the financial sector is concentrated in specific metropolitan areas. Other work finds that agglomeration can occur on a much more local scale. In particular, Rosenthal and Strange (2001, 2003) establish the level and type of agglomeration at different geographic scales, and also the measure the attenuation of these externalities within metropolitan areas. Holmes and Stevens (2002) finds evidence of differences in plant scale in areas of high concentration, suggesting production externalities act on individual establishments. A review of empirical evidence of agglomeration economies is found in Rosenthal and Strange (2004).

equilibrium following Hopenhayn (1992). Firms enter our urban economy with an initial productivity and must pay an entry cost. Productivity then evolves according to a stochastic first order Markov process. Each period firms compete in the product market, must pay a fixed cost of operating, and realize a profit. Entry, exit and relocations are dynamic and based on expectations of future productivity shocks.⁶

We characterize the optimal decision rules for firms in each location as well as those for potential entrants.⁷ Low productivity firms exit from the economy, while high productivity firms continue to operate. Relocation choices are driven by the interaction of agglomeration effects and firm productivity shocks. Due to a minimum land requirement in the production function, large firms with higher productivity shocks prefer locations with high agglomeration externalities relative to smaller, less productive firms. As a consequence, a high productivity firm that is located outside the central business district may have strong incentives to relocate to the city center.⁸ Low productivity firms leave and move to a location outside the central business district. This process gives rise to a stationary equilibrium in which firms located in the CBD are on average larger and older than firms located outside the CBD.⁹

We then develop an algorithm that can be used to estimate the parameters of our model. We focus on equilibria with entry in both locations since this is a common feature of the data. The parameters of the model are estimated using a nested fixed point algorithm. The inner loop computes the equilibrium for each parameter value, while the outer loop searches over feasible parameter values. Our simulated method of moments estimator matches the observed joint distribution of age, size and land use by location to the one predicted by our

⁶Our work is also related to Ericson and Pakes (1995) who consider the implications of oligopolistic competition on market structure. That framework is more appropriate when there are few competitors in the industry. Pakes and McGuire (1994) discuss how to solve models with oligopolistic competition. Daraszelski and Pakes (2006) provide a survey of that literature.

⁷We abstract from innovation which is discussed in detail in Klette (2004).

⁸There are some similarities with the literature that studies the sorting pattern of household in urban areas which starts with the classic papers by Alonso (1964), Mills (1967), and Muth (1969).

⁹Related to our research is also work by Melitz (2003) who studies the impact of trade on intra-industry relocations. Rossi-Hansberg and Wright (2007) examine the relationship of establishment scale and entry and exit dynamics. Finally, Combes, Duranton, Gobillon, Puga, and Roux (2010) distinguish between selection effects and productivity externalities by estimating productivity distributions across cities.

model.¹⁰

We implement the estimator using unique data collected by Dun and Bradstreet for the Pittsburgh metropolitan area. U.S. cities often act as a hub for services for a larger region. We, therefore, focus on locational choices within the service sector excluding industries in which proximity to the consumer is a key factor for firm location. The data suggest that firms located in the city are older and larger than firms located in the rest of the metro area. As a consequence they use more land and labor in the production process. However, they face higher rental rates for land and office space. Thus, they operate with a higher employee per land ratio. We find that our model explains these observed features of the data reasonably well. Our estimates imply that agglomeration externalities increase the productivity of firms by one to two percent. Economic policies that subsidize firm relocations can potentially have large effects on economic growth and firm concentration in central business districts.

The rest of the paper is organized as follows. Section 2 describes the data set used in our application and characterizes firm sorting within one metropolitan area. Section 3 develops our stochastic, dynamic equilibrium model and discusses its properties. Section 4 describes the estimation of the parameters of our model. Section 5 presents the empirical results and discusses the policy experiments. Section 6 offers some conclusions that can be drawn from the analysis.

¹⁰In related work, Davis et al. (2009) develop a growth model in which the total factor productivity of cities depends on the density of economic activity. They estimate the magnitude of this external effect and evaluate its importance for the growth rate of consumption per capita in the U.S. Our paper is thus also related to a growing literature in industrial organization that estimates dynamic models of oligopolistic competition. See, for example, Benkard (2004), Bejari, Benkard, and Levin (2007), Aguirregabiria and Mira (2007).

2.2 Data

Our empirical application focuses on firm location choices in the City of Pittsburgh and Allegheny County.¹¹ We are interested in characterizing the observed sorting of establishments by age, employment, and facility size.¹² We focus on service industries, given that there is strong evidence that large U.S. cities have undergone a transformation during the past decades moving from centers of individual manufacturing sectors toward becoming hubs for service industries. Duranton and Puga (2005), for example, show evidence that cities have become more functionally specialized, with larger cities, in particular, emerging as centers for headquarters and business services. They posit that this change is primarily related to industrial structure, and a decrease in remote management costs in particular. Davis and Henderson (2008) provide further evidence that services and headquarters are indeed more concentrated in large cities relative to the entire economy, and that headquarter concentration is linked to availability of diverse services.

We exclude wholesale and retail businesses from our analysis of services since locational decisions of these businesses are primarily driven by proximity to consumers (Hotelling, 1929).¹³ For similar reasons, we also do not consider businesses in the entertainment sector. Finally, we omit businesses related to agriculture, forestry, mining and fishing for fairly obvious reasons. We thus define the service sector as consisting of businesses that operate in information, finance, real estate, professional services, management, administrative support, education, health care and related sectors.

Figure 2.1 plots the employment concentration in Allegheny County using data from the U.S. Census. Over 20 percent of employment is concentrated in three zip codes in the center of Pittsburgh which include the downtown central business district and the business

¹¹In Appendix A of the paper we show that most other large metropolitan areas in the U.S. show sorting patterns of firms that are similar to the one we find for Pittsburgh. The comparison is based on aggregate Census data while the estimation of our model uses micro level data from Dun and Bradstreet.

¹²While we use the terms "firm" and "establishment" interchangeably, our unit of analysis in the empirical section is an establishment.

¹³Following Bresnahan and Reiss (1991), there is a large literature that explains entry and exit into markets with a small number of potential entrants. Holmes (2011) has estimated a dynamic model of market penetration of Walmart.

district in Oakland which are the two significant dense commercial areas of Pittsburgh.¹⁴ We treat these locations as the CBD in estimation while all remaining places of Allegheny County are treated as the alternate location (NCBD).¹⁵

We use firm level data from Dun and Bradstreet’s Million Dollar Database to estimate our model.¹⁶ This database covers establishments in Allegheny county in 2008 and provides detailed information on establishments. The coverage is near universal compared to Census counts of establishments in the county. The database provides data on location, facility size, total employment, industry, and year established.¹⁷

We analyze the employment and facility size characteristics for different industries in the Pittsburgh area. Table 2.1 reports the total employment, the average employment and the facility space per employee for firms in and outside the CBD for selected service industries.

Table 2.1: Employment and facility size by industry in 2008

NAICS	Total Emp.	% Emp CBD	Size NCBD	Size CBD	Facility NCBD	Facility CBD
Information	16,975	25.15%	13.52	31.16	336.88	214.44
Finance	42,960	53.51%	8.55	55.66	318.28	193.59
Real Estate	18,459	17.97%	7.51	12.43	743.36	1190.21
Professional Services	64,076	32.85%	6.99	13.29	334.83	309.60
Management	2,062	11.30%	19.46	14.56	272.88	360.52
Administrative Support	41,830	14.97%	11.01	20.67	240.89	352.14
Education	52,995	42.69%	30.46	205.66	316.70	121.27
Health Care	115,048	18.12%	16.53	24.01	293.39	291.46
Total	354,405	28.66%	11.78	27.47	326.81	265.19
Note: Size is average employment. Facility is average facility size measured in square foot per employee.						

Table 2.1 shows that 28.66 percent of all employment in the service industry is located

¹⁴The zip codes are 15222, 15219, and 15213.

¹⁵None of the results reported in this paper rely on this definition of the alternate location. We can, for example, omit those parts of Allegheny county that have little economic development and obtain similar results regarding firm sorting.

¹⁶Information on Dun and Bradstreet data is available on-line at <http://www.dnbmdd.com/>

¹⁷While most of the data are complete, the year established field, which is used to determine age of establishments, is only available for 52.5 percent of the observations. However, we find little evidence that the missing data field is systematically correlated with other observable data. We, therefore, treat observations without age of establishment as missing at random.

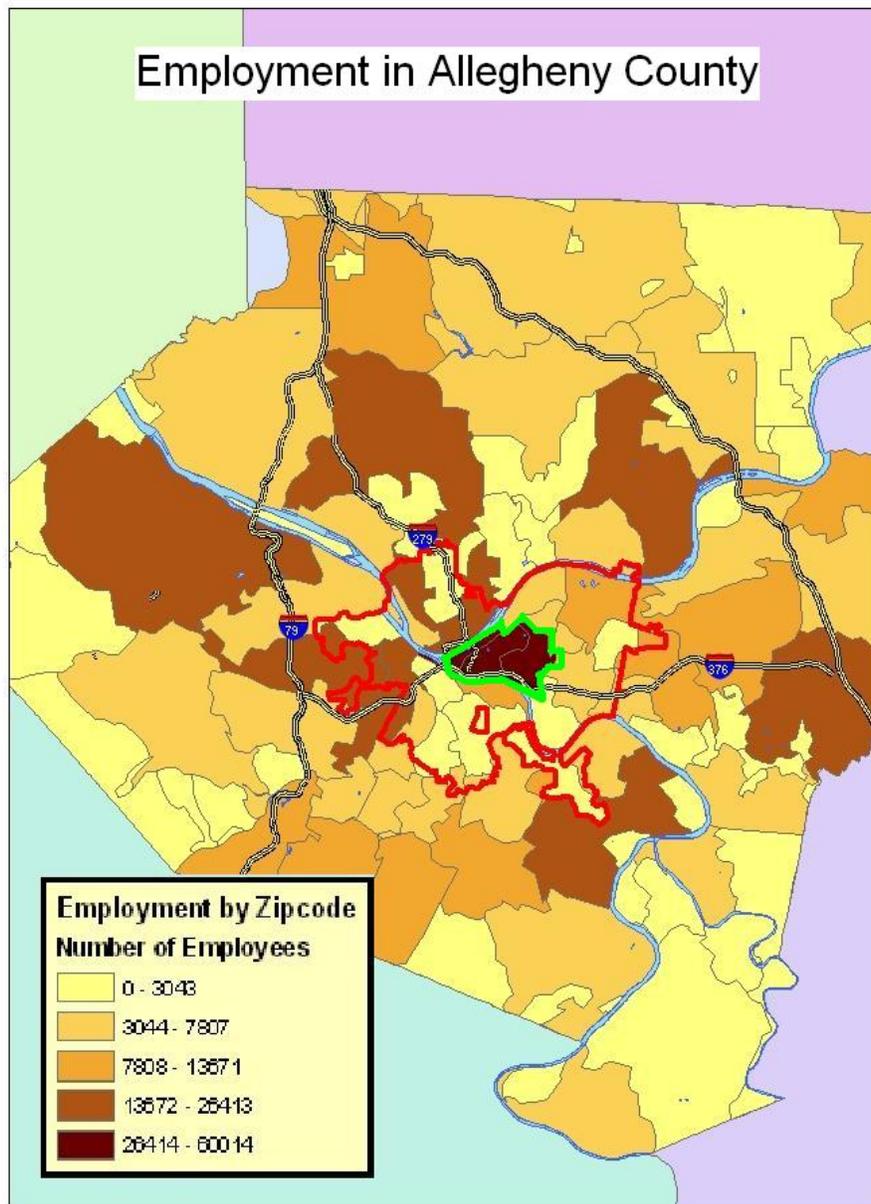


Figure 2.1: Distribution of Employment in Allegheny County

in the CBD or 13.43 percent of all our firms. However, the three zip codes that comprise the city account for a less than one percent of all the land in Allegheny county. We find that finance, education, and professional services are the industries that are most heavily concentrated in the CBD.

Comparing firms that are located inside the CBD with firms that are outside the CBD, we find some important patterns that hold for all service industries. The average employment size of establishments is larger in the central business district. The average establishment in the CBD employs 27 persons while the average firm outside the CBD has only 12 employees. However, rents for office space are higher in the CBD. As a consequence firms located in the CBD only use 265 square foot per employee while firms outside the CBD use 327.

To get some additional insights into the firm sorting process, we need to look at the full distribution of firms by location. Table 2.2 reports a number of percentiles of the age, facility size, and employment size distribution by location. Moreover, Dun and Bradstreet also report revenue estimates for each firm in the sample. These estimates must be interpreted with caution since they are likely to contain some measurement error. Nevertheless, we can use these estimates to compare output of firms across locations.

Table 2.2 reveals a number of important facts that characterize sorting of firms across locations. Firms in the CBD not only employ more workers and operate in larger facilities as we have seen above. They are also older and have a higher output per employee. The later fact is consistent with the notion that firms in the CBD may have higher productivity levels than firms located outside the CBD. Table 2.2 also shows that there are significant differences among firms in the right tail of the distribution. Looking at the 90th and higher percentiles, we find large differences between firms inside and outside of the CBD.

One potential concern of the analysis above is that differences between firms located inside and outside the CBD may be due to aggregation bias. In particular these differences could just reflect differences of sorting across different industries. In the lower panels of Table 2.2, we, therefore, report the same statistics for two industries. The middle panel

Table 2.2: Sorting of Firms by Location and Industry in 2008

All Service Industries								
	CBD				Outside CBD			
percentile	age	employ	facility	rev/emp	age	employ	facility	rev/emp
10th	6	1	890	42,500	4	1	780	28,000
25th	12	2	1,400	60,000	10	1	1,000	44,444
50th	20	5	2,500	75,000	18	3	1,700	67,647
75th	34	14	4,700	93,750	28	7	3,000	85,000
90th	53	40	11,100	185,714	41	20	5,800	127,500
95th	72	78	23,000	309,524	53	39	10,000	210,000
99th	119	460	230,000	2,000,000	96	150	50,000	892,870
Information Technology								
	CBD				Outside CBD			
percentile	age	employ	facility	rev/emp	age	employ	facility	rev/emp
10th	2	1	790	33,333	4	1	760	25,714
25th	6	3	1,600	60,000	7	1	990	52,000
50th	14	7	3,400	80,000	13	2	1,500	70,000
75th	28	20	4,800	110,000	23	6	3,000	90,000
90th	55	60	13,000	160,000	37	17	6,300	117,143
95th	90	158	20,000	250,000	56	33	11,000	182,442
99th	156	218	172,000	4,200,000	109	100	35,000	538,462
Financial Services								
	CBD				Outside CBD			
percentile	age	employ	facility	rev/emp	age	employ	facility	revenues/emp
10th	6	2	910	78,333	4	1	750	76,923
25th	12	3	1,500	93,750	9	2	1,200	85,000
50th	23	7	2,800	116,666	17	3	1,600	100,000
75th	39	17	5,500	190,000	31	6	2,300	127,500
90th	70	49	14,000	383,333	53	14	3,900	203,333
95th	79	200	34,900	725,163	72	25	6,500	400,000
99th	145	2,034	344,600	22,612,458	107	60	20,000	2,500,000

reports the results for the information technology sector which is an "average" service industry in terms of its concentration of employment in the CBD. The lower panel reports the statistics for the finance industry which is the most heavily concentrated industry in our sample. We find that the qualitative differences between firms located inside and outside the CBD are not driven by aggregation across firms in the different service industries. In anything, the differences in the financial service industry are more pronounced than the differences in sample of all service industries.

2.3 A Dynamic Model of Firm Location within an Urban Area

2.3.1 Technologies and Markets

We consider a model with two locations, denoted by $j = 1, 2$. There is a continuum of firms that produce a single output good and compete in the product market. In each period a firm chooses to stay where it is, relocate to the other location, or shutdown. Firms are heterogeneous and productivity evolves according to a stochastic law of motion.

Assumption 2.1 *In each period a firm is subject to an exogenous probability of exiting. We denote by ξ the complement probability of a firm surviving into the next period. If the firm survives, it draws a new productivity shock, φ' each time period. The productivity shock evolves over time according to a Markov process with a conditional distribution $F(\varphi'|\varphi)$.*

In our parametrized model, we assume that the logarithm of the productivity shock follows an AR(1) process, i.e. $\log(\varphi)' = \rho \log(\varphi) + \varepsilon'$, where ρ is the correlation coefficient and ε is a normally distributed random variable with mean μ_ε and variance σ_ε^2 .

Each firm produces a single output good using labor and land as input factors. The technology that is available to the firms in the economy satisfies the following assumption.

Assumption 2.2 *The production function of a firm in location j can then be written as:*

$$q = f(\varphi, n, l; e_j) \tag{1}$$

where q is output, n is labor, l is land, and e_j is the agglomeration externality in location j . The production function satisfies standard regularity conditions.

Rosenthal and Strange (2003) suggest that the externality acts as a multiplier on the production function. We use a standard Cobb-Douglas function with parameters α and γ in our computational model, $q = \varphi e_j n^\alpha (l - \bar{l})^\gamma$. Note that \bar{l} is a minimum amount of land required for production. Since $r_j \bar{l}_j$ can also be interpreted as fixed costs, this specification implies that fixed cost vary by location.

The agglomeration externality arise due to a high concentration of firms operating in the same location.

Assumption 2.3 *The agglomeration externality can be written as*

$$e_j = \Theta(L_j, N_j, S_j) \tag{2}$$

where N_j and L_j are aggregate measures of labor and land respectively, and S_j is a measure of the mass of firms in location j . The function Θ is such that $\Theta_L < 0$, $\Theta_N > 0$, and $\Theta_S > 0$.

Following Lucas and Rossi-Hansberg (2002), we assume in our computational model that

$$e_j = \left(\frac{N_j}{L_j - S_j \bar{l}} \right)^\theta \tag{3}$$

If $\theta > 0$, the externality is an increasing function of a measure of concentration of economic activity in a location j . This measure is represented by the ratio of the total number of workers and the amount of land used in production over and above the minimum land requirement.

The urban economy is part of a larger economic system which determines output prices and wages.¹⁸

Assumption 2.4 *Output prices, p , and wages, w , are constant and determined exogenously.*

Rental prices, r_j , however, are equilibrium outcomes. The supply of land is determined by an inverse land supply function in each location.

Assumption 2.5 *The inverse land supply function is given by:*

$$r_j = r_j(L_j), \quad j = 1, 2 \tag{4}$$

The inverse supply function is increasing in the amount of land denoted by L_j .

In the computational analysis, we adopt an iso-elastic functional form: $r_j = A_j L_j^\delta$, $j = 1, 2$, where A_j and δ are parameters. Since rental prices for land must be higher in equilibrium in locations with high externalities, the agglomeration externality is, at least, partially capitalized in land rents.

We can break down the decision problem of firms into a static and a dynamic problem. First, consider the static part of the decision problem that a firm has to solve each period. This problem arises because firms compete in the product market each period.

Assumption 2.6 *The product market is competitive and firms behave as price takers. Firms make decisions on land and labor usage after they have observed their productivity shock, φ , for that period.*

Let π_j denote a firm's one period profit in location j . The static profit maximization problem can be written as:

$$\{n, l\} = \arg \max_{\{n, l\}} \pi_j(n, l; \varphi), \tag{5}$$

¹⁸it is straight forward to endogenize wages by adding a local labor market to our model.

where the profit function is given by:

$$\pi_j(n, l; \varphi) = p f(\varphi, n, l; e_j) - w n - r_j l - c_f. \quad (6)$$

The parameter c_f denotes a fixed cost of operation independent of location. Solving this problem we obtain the demand for inputs as a function of φ , denoted by $n_j(\varphi)$ and $l_j(\varphi)$, as well as an indirect profit function, denoted by $\pi_j(\varphi)$.¹⁹

Let μ_j denote the measure of firms located in j . The mass of firms located in j , denoted by S_j , is given by the following expression:

$$S_j = \int \mu_j(d\varphi) \quad (7)$$

Given the static choices for land and labor use for each firm, we can also calculate the aggregate levels of land and labor:

$$L_j = \int l_j(\varphi) \mu_j(d\varphi), \quad (8)$$

$$N_j = \int n_j(\varphi) \mu_j(d\varphi) \quad (9)$$

After choosing labor and land inputs, each firm faces the (dynamic) decision of whether to stay in its current location, move to the other location, or shut down. The following Bellman equations formalize the decision problem of a firm that begins the period in location j with a productivity shock φ :

$$V_1(\varphi) = \pi_1(\varphi) + \beta \xi \max \left\{ 0, \int V_1(\varphi') F(d\varphi'|\varphi), \int V_2(\varphi') F(d\varphi'|\varphi) - c_r(\varphi) \right\} \quad (10)$$

$$V_2(\varphi) = \pi_2(\varphi) + \beta \xi \max \left\{ 0, \int V_2(\varphi') F(d\varphi'|\varphi), \int V_1(\varphi') F(d\varphi'|\varphi) - c_r(\varphi) \right\} \quad (11)$$

where β is the discount factor, $c_r(\varphi)$ is the cost of relocating from one location to another. We explore different specifications in the quantitative analysis. One specification assumes

¹⁹Note that the sub-index j summarizes the dependence of the profit and input demand functions on location j 's rent and externality.

that relocation costs depend on the size of the firm, $q(\varphi)$.

Solving the dynamic decision problem above implies decision rules of the following form for firms currently in location j :

$$x_j(\varphi) = \begin{cases} 0 & \text{if firm exits in next period} \\ 1 & \text{if firm chooses location 1 in next period} \\ 2 & \text{if firm chooses location 2 in next period} \end{cases} \quad (12)$$

To close the model, we need to specify the process of entry.

Assumption 2.7 *Firms can enter into both locations. All prospective entrants are ex-ante identical. Upon entering a new firm incurs a cost c_{ej} and draws a productivity shock φ from a distribution $\nu(\varphi)$.*

Note that we allow the entry cost to vary by location. In our parametrized model the entrant distribution is assumed to be log-normal with parameters μ_{ent} and σ_{ent}^2 . These assumptions guarantee that the expected discounted profits of a prospective firm are always less or equal than the entry cost:

$$c_{ej} \geq \int V_j(\varphi) \nu(d\varphi), j = 1, 2 \quad (13)$$

If there is positive entry of firms, then this condition holds with equality.

2.3.2 Equilibrium

We are now in a position to define a stationary equilibrium to our economy.

Definition 2.1 *A stationary equilibrium for this economy consists of rents, r_j^* , masses of entrants, M_j^* , stationary distributions of firms, $\mu_j^*(\varphi)$, externalities, e_j^* , land demand functions, $l_j^*(\varphi)$, labor demand functions, $n_j^*(\varphi)$, value functions, $V_j^*(\varphi)$, and decision rules, $x_j^*(\varphi)$, for each location $j = 1, 2$, such that:*

1. *The decision rules (12) for a firm's location are optimal, in the sense that they max-*

imize the right-hand side of equations (10).

2. The decision rules for labor and land inputs solve the firm's static problem in (5).
3. The free entry conditions (13) are satisfied in each location, with equality if $M_j^* > 0$.
4. The market for land clears in each location consistent with equation (4).
5. The mass of firms in each location is given by equation (7).
6. The externalities are consistent with (2)
7. The distributions of firms μ_j^* are stationary in each location and consistent with firms' decision rules.

Any stationary equilibrium to our model can be characterized by vector of equilibrium values for rents, mass of entrants, and externalities in each location $(r_1, r_2, M_1, M_2, e_1, e_2)$. Finding an equilibrium for this model is equivalent to the problem of finding the root of a nonlinear system of equations with six equations. For any vector $(r_1, r_2, M_1, M_2, e_1, e_2)$, we can

1. solve the firms' static profit maximization problem and obtain land demand, labor demand, and the indirect profit functions for each location;
2. solve the dynamic programming problem in equations (10) and obtain the optimal decision rules;
3. use the initial mass of entrants in each location and simulate the economy forward until the distribution of firms, μ_j , converges to a stationary distribution;
4. calculate the aggregates land and labor demands, as well as the land supply in the economy;
5. check whether market clearing conditions and the equations that define the mass of firms and the externalities in each location are satisfied.

If the equilibrium conditions are not satisfied, we update the vector of scalars and repeat the process until all of the conditions for equilibrium are satisfied. If this algorithm converges, we have computed an equilibrium of the model.

Note that the mapping described above is not a simple contraction mapping. As a consequence we cannot apply standard techniques and proof of existence of equilibrium. In Appendix B of the paper, we provide a proof of existence of equilibrium for a simplified version of our model in which firm productivity is constant across time. Moreover, we have computed equilibria for a large number of different specifications of our general model. We, therefore, conclude that equilibria exist for reasonable parameterizations of the model.

The task of computing an equilibrium can be simplified by exploiting some properties of the parametrization used in our computational model. The static first order condition that determines that ratio of land and labor inputs is given by:

$$\frac{n}{l - \bar{l}} = \frac{\alpha r_j}{\gamma w} \quad (14)$$

Notice that the ratio in this equation is the same for all firms in the same location j . Aggregating over all firms in such location, we obtain that:

$$\frac{N_j}{L_j - S_j \bar{l}} = \frac{\alpha r_j}{\gamma w} \quad (15)$$

Equation (15) then implies an expression linking the externality, e_j in each location to that location's rent, r_j . We can, therefore, solve the Bellman equations without knowing the aggregate levels of land and labor. As a consequence we can characterize equilibrium rent values solely based on the free entry conditions expected values functions, which can be written as:

$$EV_1(r_1, r_2) = \int V_1(\varphi) d\nu(\varphi) \quad (16)$$

$$EV_2(r_1, r_2) = \int V_2(\varphi) d\nu(\varphi) \quad (17)$$

The entry condition for location one then defines a mapping $r_1 = \Gamma_1(r_2)$, i.e. for given r_2 , $\Gamma_1(r_2)$ is the value of r_1 such that $EV_1(r_1, r_2) = c_e$. Similarly, we can define a mapping $r_1 = \Gamma_2(r_2)$ for location two. These two mappings then effectively define the set of rent pairs $\{r_1, r_2\}$, such that the two free entry conditions are satisfied with equality.

The non-linearity of the model implies that $\Gamma_1(r_2)$ and $\Gamma_2(r_2)$ can intersect multiple times. As a consequence, there may be more than one possible candidate values for equilibria with entry in both locations.²⁰

Next, define the ratio of entrants in the two locations as $m = \frac{M_1}{M_2}$ and the distribution of firms standardized by the mass of entrants in locations 2 as,

$$\hat{\mu}_j = \frac{\mu_j}{M_2}. \quad (18)$$

The standardized stationary distributions satisfy

$$\begin{aligned} \int_0^{\varphi'} \hat{\mu}_1(dx) &= \xi \int F(\varphi'|\varphi) 1\{x_1(\varphi) = 1\} \hat{\mu}_1(d\varphi) \\ &+ \xi \int F(\varphi'|\varphi) 1\{x_2(\varphi) = 1\} \hat{\mu}_2(d\varphi) + m \int_0^{\varphi'} \nu(dx) \\ \int_0^{\varphi'} \hat{\mu}_2(dx) &= \xi \int F(\varphi'|\varphi) 1\{x_1(\varphi) = 2\} \hat{\mu}_1(d\varphi) \\ &+ \xi \int F(\varphi'|\varphi) 1\{x_2(\varphi) = 2\} \hat{\mu}_2(d\varphi) + \int_0^{\varphi'} \nu(dx) \end{aligned} \quad (19)$$

where $1\{x_j(\varphi) = j\}$ is an indicator function equal to 1 if x_j equals j and 0 otherwise. Given a value for m , forward iteration on these two equations yields the equilibrium standardized stationary distributions $\hat{\mu}_j$, $j = 1, 2$.

To find the equilibrium value of m , substitute the aggregate demands for land in the two locations into the inverse land supply functions and take their ratios. Given that the

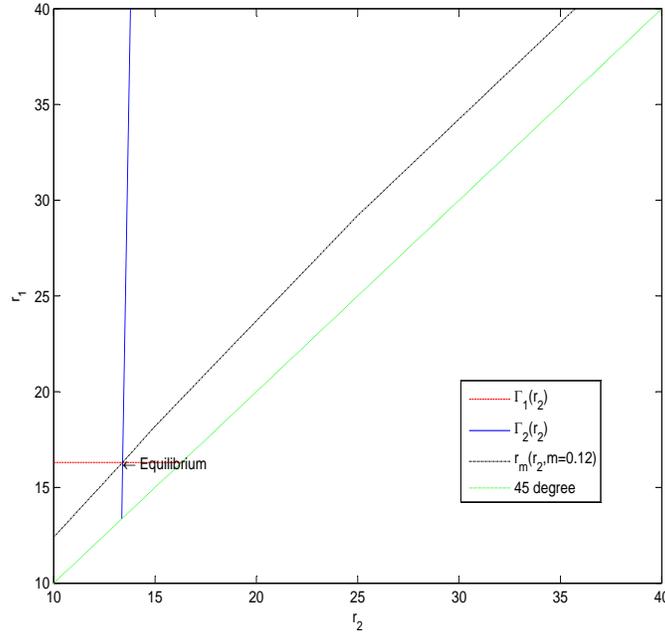
²⁰In addition to equilibria with entry in both locations, it is also possible to have equilibria in which entry only occurs in one of the two locations.

inverse elasticity, denoted by δ , is the same in both locations we then obtain:

$$\frac{r_1}{r_2} = \frac{A_1}{A_2} \left[\frac{\int l_1(\varphi) \hat{\mu}_1(d\varphi)}{\int l_2(\varphi) \hat{\mu}_2(d\varphi)} \right]^\delta. \quad (20)$$

Let $r_1 = r_m(r_2; m)$ be the value of r_1 that clears the relative land markets given r_2 and m , keeping in mind that both the labor demand functions $l_j(\varphi)$ and the masses of firms $\hat{\mu}_j$ depend on r_1 and r_2 .

Figure 2.2: Graphical Representation of Equilibrium



We can thus conclude that all rent pairs $\{r_1, r_2\}$ that are consistent with entry in both locations are characterized by the intersection of the two functions $\Gamma^j(r_2)$. In addition, we have characterized the set of rent pairs consistent with land market clearing condition, $r_m(r_2; m)$, corresponding to different values of m . By analyzing these functions all together, we can completely characterize the set of triplets $\{r_1, r_2, m\}$ consistent with equilibrium

in the economy. Figure 2.2 illustrates a (locally unique) equilibrium that arises in our model. Note that in this specification of the model firms only relocate from community 2 to community 1 in equilibrium.

Finally, the mass of entrants in location 2, M_2 , is determined by the market clearing condition for land:

$$\left(\frac{r_2}{A_2}\right)^{\frac{1}{\delta}} = M_2 \int l_2(\varphi) \hat{\mu}_2(d\varphi), \quad (21)$$

Note that M_2 can be solved for analytically.

Characterizing additional properties of the equilibrium is difficult. In Appendix B of the paper we consider a simplified version of our model in which the productivity of firms does not change over time. Under this simplifying assumption we can analytically characterize the resulting stationary distribution of firms in equilibrium. For the general version of the model, additional insights can be gained using numerical methods.

With respect to uniqueness of stationary equilibrium, there are four issues. First, as is common in multi-community models, equilibrium typically exists with communities that are ex post identical. These “non-sorting” equilibria are uninteresting and easily rejected empirically. We analyze sorting equilibria here. Second, the non-convexities in the model associated with community choice preclude use of standard techniques to establish uniqueness of sorting equilibria. Third, entry conditions may not hold with equality which can give rise to equilibria with entry in only one of the two locations. In the computational analysis, we only focus on equilibria with entry in both locations. Last, the endogeneity of the firm productivity distribution in stationary equilibrium may not be unique.

While there are several sources of potential multiplicity, we find in our computational analysis that stationary (sorting) equilibria are robust. When we perturb an equilibrium that we have computed, the algorithm converges back to the original equilibrium. These computational experiments suggest that equilibrium is at least locally unique. We do not have a formal proof of local uniqueness of the sorting equilibrium.

2.4 Estimation

Let θ denote the parameter vector of the structural model to be estimated. Given the micro data that we observe for our sample discussed in Section 2, we could, in principle, construct a Maximum Likelihood Estimator for the parameters of the model. The basic idea behind this estimator is the following. We observe firm output, labor input, land use, and the aggregates that determine the agglomeration externality for a random sample of firms in each location. We can, therefore, express the unobserved productivity, ω , as a function of the observed variables and the unobserved parameters of the model. Moreover, we can characterize this productivity conditional on age as well as relocation, entry, and exit decisions of each firm in the sample. We have seen in Section 3 that the equilibrium of our model defines a nonlinear mapping from the parameter vector θ to the stationary distribution of firm level productivities. In particular, there exist stationary densities of firm productivities conditional on location, age, and the endogenous entry, exit, and relocation decisions of firms. We can, therefore, construct a likelihood function based on these conditional densities.²¹ This likelihood estimator is difficult to implement in practice since the stationary distribution of firm productivities does not have an analytical solution. The corresponding density is hard to compute with high accuracy. Moreover, the revenue estimates provided by Dun and Bradstreet may be inaccurate, in particular for small firms.

We, therefore, adopt a simulated method of moments approach to estimate the parameters of our model that treats output as a latent variable.²² The estimation strategy relies on the idea that the structural model should replicate the observed joint empirical distribution function of age, facility size, and employment conditional on location choice.

The observed joint empirical distribution function of age, facility size, and employment conditional on location choice can be captured by moments that are based on histograms.

²¹Note that one would need to account for measurement error in output to obtain a well-behaved likelihood function.

²²Alternatively, we could estimate an auxiliary model using semi-nonparametric estimation as developed by Gallant and Nychka (1987) and then match the scoring functions of the auxiliary and the structural model (Gallant & Tauchen, 1987).

In practice, these moments are constructed by placing establishments into categories, such as firms with 5 to 8 employees, 11 to 20 years old, located in the city. These type of moments then are calculated as the percentage of firms in a given category relative to the number of establishments in the entire metropolitan area. In addition, we use the total percentage of firms in the CBD relative to the full metropolitan area.

Combine all moments used in the estimation procedure into one vector m_N and denote with $m_S(\theta)$ their simulate counterparts where S denotes the number of simulations. The orthogonality conditions are then given by

$$g_{N,S}(\theta) = m_N - m_S(\theta) \quad (22)$$

Following Hansen (1982), θ can be estimated using the following moments estimator:

$$\theta_{S,N} = \arg \min_{\theta \in \Theta} g_{S,N}(\theta)' A_N g_{S,N}(\theta) \quad (23)$$

for some positive semi-definite matrix A_N which converges in probability to A_0 . Since we can make the simulation error arbitrarily small, we suppress the dependence of our estimator on S . The estimator θ_N is a consistent estimator of θ_0 and:

$$N^{1/2} (\theta_N - \theta_0) d \rightarrow N(0, (\tilde{A}_0 D_0)^{-1} \tilde{A}_0' V \tilde{A}_0 (\tilde{A}_0 D_0)^{-1'}) \quad (24)$$

where $\tilde{A}_0 = D_0' A_0$, $D_0 = E[\partial m(\theta) / \partial \theta_0]$ and V is asymptotic covariance matrix of the vector of sample moments, and θ_0 denotes the parameter of the data generating process. The most efficient estimator is obtained by setting $A_N = V_N^{-1}$. In this case:

$$N^{1/2} (\theta_N - \theta_0) d \rightarrow N(0, (D_0' V^{-1} D_0)^{-1}) \quad (25)$$

Furthermore, standard J-statistics can be used for hypothesis and specification tests.²³

²³Strictly speaking, one would need to correct for the sampling error induced into the estimation procedure by the simulations. However, if the number of simulations is large, these errors will be negligible. For a review see Gourieroux and Monfort (1992).

2.5 Some Preliminary Estimation Results

Table 2.3 reports the parameter estimates and the estimated standard errors for a variety of model specifications. Wages in our model are equal to \$48,661 which corresponds the average yearly income in the financial/service sectors, based on census business patterns data. We also set α , the labor share of to be equal to 0.65.²⁴ The facility supply elasticity, denoted by δ , is set equal to 0.2.²⁵ Last, we set the exogenous exit probability to 0 (or $\xi = 1$) and the discount factor equal to 0.95, taking a year as the relevant unit of time.

We consider three different model specifications. The first model reported in column I of Table 2.3 assumes that relocation costs are constant and equal \$200,000. The second specification reported in column II assumes that relocation costs are proportional to output. This specification accounts for the fact that large firms face higher relocation costs than small firms. The third specification uses a concave relocation cost function.

First, consider the version of the model with relocation costs equal to \$200,000. We find that the estimate of the land share parameter is 0.0884. The parameter estimate of the agglomeration externality is 0.102. We have seen before that the restriction that $\theta > \gamma$ is necessary to get an equilibrium sorting pattern in which high productivity firms prefer locations with high agglomeration externalities. These parameters are estimated with high precision.

The fixed costs of operation are \$52,000, or a quarter of the costs of relocating to a different community. Entry costs differ by location and are estimated to be \$764,000 and \$786,000, respectively. The minimum land requirement is approximately 1210 square foot with an estimate standard error of 344. The productivity shocks are highly correlated across time. The point estimate of 0.977 is consistent with previous estimates in the literature Hopenhayn and Rogerson (1993).

²⁴Estimates about the land share are reported in Deckle and Eaton (1993), Adsera (2000), and Caselli and Coleman (2001).

²⁵Estimates vary for this rent elasticity of supply for office space, but are generally accepted to be greater than unity. See Wheaton (1999) for discussion for estimates.

Table 2.3: Parameter Estimates

	I	II	III
c_r	200,000	.01 * q	$q^{.73}$
ρ	0.9773 (0.001)	0.9772	0.9770
σ_ϵ	0.137 (0.001)	0.132	0.137
μ_ϵ	0.239 (0.001)	0.240	0.243
μ_{ent}	11.67 (0.001)	11.63	11.62
σ_{ent}	0.230 (0.001)	0.306	0.343
l_{min}	1210 (344)	1058	1120
θ	0.102 (0.001)	0.104	0.110
γ	0.088 (0.001)	0.090	0.093
c_f	51900 (2880)	52590	59831
ce_1	764000 (12900)	780000	844000
ce_2	786000 (13000)	785000	849000
A_1/A_2	1.6327 (0.189)	1.8411	1.7830

The parameter estimates are similar for the other two specifications of the model reported in columns II and III. Fixed costs and relocation costs are higher in these specifications while minimum land requirement and thus the location specific fixed costs are lower. However, the three specifications imply equilibria that differ in some qualitative and quantitative features. Table 2.4 reports rental price ratios and relocation patterns implied by the three model specifications.

Table 2.4: Rental Price Ratios and Relocation

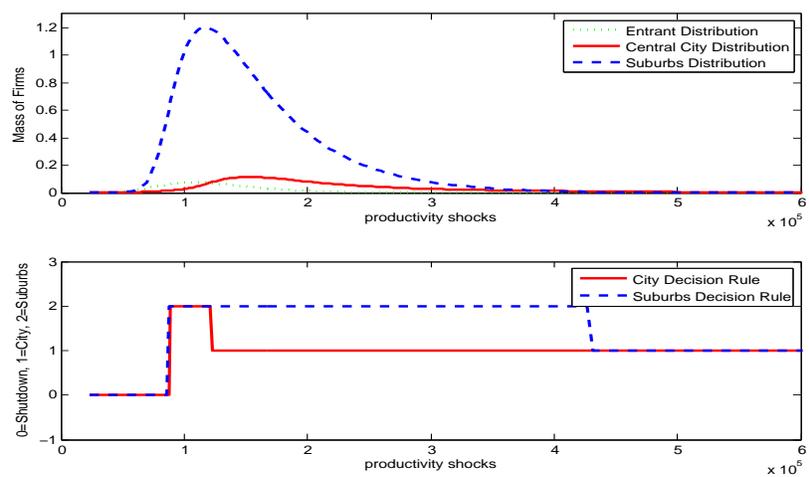
	I	II	III
c_r	200,000	.01 * q	$q^{.73}$
ratio of rents	1.216	1.228	1.114
Reloc. cost per emp. (\$)	732.18	813.67	1214.12
% est. Move (2 to 1)	0.03 %	0.09 %	0.07 %
% emp Move (2 to 1)	1.02 %	1.21 %	1.36 %
%t est. Move (1 to 2)	0.00 %	1.68 %	1.11 %
% emp. Move (1 to 2)	0.00 %	0.53 %	0.21 %

The equilibrium associated with the first model specification implies that the rental rate for office in the CBD is approximately 21 percent higher than the rate outside the CBD. This estimated price ratio is similar to the one reported by the Building Owners and Managers Association.²⁶ The price ratio along with the estimate of the externality parameter, θ , implies that firms located in the CBD receive a 2.02 percent productivity gain over firms located elsewhere due to the local agglomeration externality. For the third model specification the price ratio is smaller and we obtain a 1.2 percent productivity gain. Overall, the magnitude of these productivity gains are small, but not irrelevant.

The upper panel of Figure 2.3 plots the stationary distribution of firms in both locations as well as the distribution of entrants for specification III. Note that neither of these distributions must integrate to one since the mass of firms and entrants are equilibrium outcomes. The lower panel of Figure 2.3 plots the optimal decision rules. The equilibrium

²⁶The Building Owners and Managers Association collects information on expenses and income for office space throughout North America. They report a rent ratio of 1.22 for suburban and CBD office space for the United States. See www.boma.org for more information.

Figure 2.3: Stationary Distributions and Decision Rules



implies that firms with high productivity shocks relocate to the CBD while low productivity firms leave the CBD to operate in cheaper locations.

Table 2.4 quantifies the impact of relocation. Specification 1 of our model implies that there is only relocation of large firms from outside the CBD into the CBD. While only a small fraction of firms move to the CBD, they account for approximately one percent of employment in the metro area. In specifications II and III, the equilibrium implies that small firms will relocate from the CBD to locations outside the CBD. Approximately one percent of all small firms leave the CBD per year.

Table 2.5: Age-Employment distributions of establishments by location, as a percentage of total establishments in the entire county (computational moments in parenthesis)

Inside CBD					
		Age			
		1 to 10	11 to 20	21 to 30	> 30
Employment	1 to 3	1.48 (2.42)	1.71 (1.13)	0.96 (0.36)	1.17 (1.68)
	4 to 8	0.59 (0.94)	1.10 (0.56)	0.56 (0.36)	0.93 (1.07)
	9 to 16	0.33 (0.27)	0.56 (0.25)	0.48 (0.18)	0.64 (0.62)
	> 16	0.41 (0.12)	0.64 (0.22)	0.63 (0.25)	1.24 (1.18)
	Outside CBD				
		Age			
		1 to 10	11 to 20	21 to 30	> 30
Employment	1 to 3	16.25 (20.99)	16.00 (10.25)	9.48 (5.97)	8.17 (11.77)
	4 to 8	3.59 (7.86)	5.89 (4.82)	4.64 (3.11)	4.44 (6.56)
	9 to 16	1.44 (2.25)	2.69 (2.07)	1.81 (1.50)	2.31 (3.38)
	> 16	1.44 (0.95)	2.84 (1.61)	2.04 (1.42)	3.50 (3.57)

Finally, we evaluate the within sample fit of our model. Table 2.5 reports the distribution of firms by age and employment size for our sample and the one predicted by our model for specification I of the model. We have seen before that the data suggest that firms located

in the city are older and larger than firms located in the rest of the metro area. Moreover, the firms in the city have more employees holding age constant. Overall, our model fits this feature of the data reasonably well.

Table 2.6: Age-Facility Size establishment distributions by location as percentage of total establishments in the county, (computational moments in parenthesis)

Inside CBD					
		Age			
		1 to 10	11 to 20	21 to 30	> 30
Facility (sq ft)	1 to 1250	0.82 (2.06)	0.73 (0.95)	0.44 (0.54)	0.65 (1.38)
	1251 to 2150	0.64 (0.82)	1.18 (0.43)	0.59 (0.26)	0.88 (0.43)
	2151 to 3850	0.75 (0.48)	1.13 (0.32)	0.70 (0.21)	0.82 (0.64)
	> 3850	0.60 (0.38)	0.98 (0.47)	0.91 (0.43)	1.63 (1.81)
Outside CBD					
		Age			
		1 to 10	11 to 20	21 to 30	> 30
Facility (sq ft)	1 to 1250	9.57 (15.39)	9.76 (7.42)	5.51 (4.26)	4.20 (8.31)
	1251 to 2150	7.08 (6.99)	7.56 (3.58)	4.86 (2.17)	4.74 (3.40)
	2151 to 3850	3.81 (5.53)	6.01 (3.38)	4.14 (2.18)	4.18 (4.58)
	> 3850	2.30 (4.14)	4.09 (4.38)	3.45 (3.40)	5.13 (7.97)

Table 2.6 focuses on the age-facility size establishment distributions. Employment and facility size are highly correlated in the data as predicted by our model. As a consequence, Table 2.6 reinforces our previous findings about the model fit. Firms in the city face higher rental rates for land and office space. As a consequence they operate with a higher employee per land ratios. Our model explains these features of the data well.

Our analysis has some important policy implications. Relocation costs prevent establishments from moving because the gains for the individual firm are not worth the moving cost. However, this decision may not be efficient since firms ignore the external benefits of

density and agglomeration to other firms when making locational decisions. A relocation subsidy may help firms to sort more efficiently in the urban economy. However, it is also possible that relocation policies are not desirable. In large metropolitan areas, there are often many independent communities that compete among each other to attract business using targeted subsidies. It is not obvious that this type of tax and subsidy competition among communities increases economic welfare.

To evaluate these types of policies, we need to measure economic welfare. In our model, establishments have zero expected profits. Hence, the most useful measure of welfare in this economy is surplus that arises to land owners in equilibrium. This surplus can be measured as the area between the rent and the land supply curve:

$$Surplus = \int_0^{L_1^*} (r_1^* - A_1 L^\delta) dL + \int_0^{L_2^*} (r_2^* - A_2 L^\delta) dL. \quad (26)$$

– to be continued –

2.6 Conclusions

We have developed a new dynamic general equilibrium model to explain firm entry, exit, and relocation decisions in an urban economy with multiple locations. We have characterized the stationary distribution of firms that arises in equilibrium. The parameters of the model can be estimated using a nested fixed point algorithm by matching the observed distribution of firms by location and the one implied by our model. We have implemented the estimator using unique data collected by Dun and Bradstreet for the Pittsburgh metropolitan area. Firms located in the central business district are older and larger than firms located outside the urban core. They use more land and labor in the production process. However, they face higher rental rates for office space which implies that they operate with a higher employee per land ratio. Our estimates imply that agglomeration externalities increase the productivity of firms by one to two percent. Economic policies that subsidize firm relocations can potentially have large effects on economic growth and firm concentration in central

business districts. We view the findings of this paper as promising for future research. Our model can also be used to study relocations of firms across metropolitan area. Moreover, we extend the modeling framework in a number of useful directions to analyze investment and innovation decisions.

2.A Appendix: Firm Location Choices in a Sample of Large U.S. Cities

To get some additional quantitative insights into firms sorting behavior, we collected Census data for a number of metro areas. We define a business district within the metropolitan area as those zip codes within a city that have a high density of firms signifying local agglomeration. To make this concept operational, we use an employment density of at least 10,000 employees per square mile. These locations need not be contiguous, as some metropolitan areas exhibit multiple dense business districts.

Table 2.7 shows the concentration of employment in dense business districts for a sample of U.S. cities. First, we report statistics using all firms that are located in the metro area. We find that there is a significant amount of heterogeneity among the cities in our sample. There are some cities such as Phoenix and Hartford where employment is not concentrated in dense business districts. Most larger cities in the U.S. such as Los Angeles, Chicago, Boston, Washington, Philadelphia, and Houston have a significant fraction of firms located in high density central business districts. This finding is also true for a variety of mid-sized cities such as Pittsburgh and Seattle. Focusing on the differences between firms located in and out of the CBD, we find firms in the CBD are larger than the MSA average. This indicates that they have higher levels of productivity. This finding is common among all cities in our sample. In addition, firms in the service sector are more concentrated in the CBD compared to firms in general, suggesting that service oriented firms benefit more from local agglomeration than other sectors.

Table 2.7: Concentration of employment in dense business districts

MSA	Total Emp. Outside CBD	Total Emp. in CBD	Avg. Emp. outside CBD	Avg. Emp. in CBD	% Services outside CBD*	% Services in CBD**
Atlanta	1,115,398	229,002	15.79	29.25	45.24%	63.31%
Boston	1,728,075	531,349	15.66	39.01	41.99%	59.90%
Chicago	3,070,387	528,529	15.86	24.47	41.85%	66.50%
Columbus	705,534	63,278	18.69	23.73	42.88%	58.64%
Hartford	499,718	18,783	17.26	26.95	40.31%	61.41%
Houston	1,720,625	286,574	16.38	28.47	42.86%	65.51%
Jacksonville	491,959	24,315	15.24	25.38	43.09%	66.28%
Los Angeles	4,257,269	974,693	15.02	19.39	44.16%	52.39%
Philadelphia	1,921,626	196,428	15.91	27.66	43.99%	55.74%
Phoenix	1,551,921	64,793	18.31	27.78	47.79%	71.01%
Pittsburgh	822,013	157,009	14.58	40.04	39.16%	60.90%
Salt Lake	440,239	53,086	15.22	21.08	45.64%	58.90%
San Antonio	655,740	26,572	17.21	20.49	43.22%	56.59%
Seattle	1,260,335	179,230	14.55	20.33	42.07%	58.97%
St Louis	1,253,959	84,034	16.38	42.57	41.41%	52.43%
Wash. DC	1,930,848	303,770	15.42	21.68	49.96%	60.05%

Source: 2006 Zip Code Business Patterns, U.S. Census

*Percentage of establishments outside the CBD that are in the service industries.

**Percentage of establishments in the CBD that are in the service industries.

2.B Appendix: Analytical Properties of Equilibrium

To get some additional insights into the properties of our model it is useful to simplify the structure of the model and shut down the future productivity shocks. We can then characterize the equilibrium of the model almost in closed-form.²⁷ Let us impose the following additional assumptions.

Assumption 2.8

²⁷The model cannot be entirely solved in closed form because the equilibrium r_2 has to satisfy a highly non-linear equation. Sufficient conditions on the model's parameters for r_2 to exist and be unique are imposed instead. Conditional on r_2 , everything else can be solved for analytically.

1. The shock is drawn upon entry once and for all from a uniform distribution in $[0, 1]$:

$$\nu(\varphi) = 1 \text{ for } \varphi \in [0, 1]. \quad (27)$$

2. There are no fixed cost of operation: $c_f = 0$.

3. Importance of externality: $\theta = 1 - \alpha > \gamma$

Let 1 denote the high rent location and 2 the low rent one (1=city, 2=suburb). We show how to construct a unique equilibrium in which $r_1 > r_2$ and firms move from location 2 to location 1, but not vice versa. Firms who enter in location 1 stay there all the time or exit.

First note that under assumptions 2 and 3 above the indirect profit functions can be written as:

$$\pi_j(\varphi) = r_j \left(\Delta \varphi^\eta - \bar{l} \right), j = 1, 2, \quad (28)$$

where $\Delta > 0$ and $\eta > 1$ are known functions of the parameters of the model. Consider location in the city. We have the following result.

Proposition 2.1 *If $r_1 > r_2$,*

a) then firms in location 1 follow a simple cut-off rule. Firms below a threshold φ_l exit while firms above the threshold stay in location 1 forever. The cut-off is defined as:

$$\varphi_l = \left(\frac{\bar{l}}{\Delta} \right)^{\frac{1}{\eta}}. \quad (29)$$

b) then firms in location 2 follow a simple cut-off rule. Firms below the threshold φ_l exit, firms with shocks between φ_l and φ_h stay in location 2, and firms with shocks larger than φ_h move to location 1. The cut-off φ_h is defined as:

$$\varphi_h = \left(\frac{\bar{l}}{\Delta} + \frac{c_r(1 - \beta\xi)}{\Delta(r_1 - r_2)} \right)^{\frac{1}{\eta}}. \quad (30)$$

Proof:

a) Note that static firm profits are monotonically increasing in φ . Define φ_l such that $\pi_1(\varphi_l) = 0$. Then firms with $\varphi < \varphi_l$ exit immediately. It is straight forward to show that

$$V_1(\varphi_l) = \pi_1(\varphi_l) + \beta\xi \max\{0, V_1(\varphi_l), V_2(\varphi_l) - c_r\} = \pi_1(\varphi_l) = 0 \quad (31)$$

where the second equality follows from the fact that the productivity cut-off for switching to location 2 is less than cut-off for exit if $r_1 > r_2$ as assumed. Firms with $\varphi > \varphi_l$ stay in location 1 as long as they survive the exogenous destruction shock ξ . Their payoffs are:

$$V_1(\varphi) = \frac{\pi_1(\varphi)}{1 - \beta\xi} > 0 \quad (32)$$

b) Next consider the decision rule of firms located in the suburb. Firms with $\varphi < \varphi_l$ exit immediately:²⁸

$$V_2(\varphi_l) = 0 \quad (33)$$

Firms with shocks in (φ_l, φ_h) stay in 2 forever (as long as they survive the exogenous destruction shock). Firms with high shock move to 1. The indifference condition for staying vs moving is:

$$\frac{\pi_2(\varphi_h)}{1 - \beta\xi} = \pi_2(\varphi_h) + \beta\xi (V_1(\varphi_h) - c_r). \quad (34)$$

This equation defines the cut-off value φ_h . The lemma then follows from the result that benefits of switching to location 1 monotonically increase with φ . Q.E.D.

Next we consider the free entry conditions and show that these conditions determine the rents in both locations. We have the following result:

Proposition 2.2 *There is at most one set of rental rates (r_1, r_2) that are consistent with the entry in both locations. Conditions on the parameter values guarantee existence of (r_1, r_2) .*

²⁸Note that equation (28) implies that z_l does not depend on the location.

Proof (of uniqueness):

First consider the free entry condition in location 1 which is given by

$$\int V_1(\varphi) \nu(\varphi) d\varphi = c_e. \quad (35)$$

Substituting in our optimal decision rule and simplifying we obtain the equilibrium rent in location 1:

$$r_1 = \frac{c_e (1 - \beta\xi) (\eta + 1)}{\Delta (1 - \beta\xi\varphi_l^{\eta+1}) - \bar{l} (1 - \beta\xi\varphi_l) (\eta + 1)}. \quad (36)$$

Free entry in location 2 requires:

$$\int V_2(\varphi) \nu(\varphi) d\varphi = c_e. \quad (37)$$

Replacing the value function in location 2 and taking into account the definition of φ_h in (30) this equation simplifies to:

$$\eta\varphi_h^{1+\eta} - (1 + \eta)\varphi_h^\eta - K = 0, \quad (38)$$

where K represents a non-positive combination of the parameters and is defined in the appendix. The left hand side of this equation is positive when $K = 0$ and has a negative first derivative. Thus, if a solution for φ_h exists it must be unique. In turn, φ_h is monotonically related to r_2 by equation (30):

$$r_2 = r_1 - \frac{c_r (1 - \beta\xi)}{\Delta\varphi_h^\eta - \bar{l}}. \quad (39)$$

Thus, if the solution φ_h to equation (38) is unique, the equilibrium value of r_2 is also unique. The appendix provides sufficient conditions on the parameters for this solution to exist. Q.E.D.

Next we characterize the equilibrium distribution of firms in each location.

Proposition 2.3 *For each value of M_2 , there exists a unique stationary equilibrium distri-*

butions of firms in each location.

Proof:

Without loss of generality, let us normalize the model so that entry in location 2 is always equal to $M_2 = 1$. This implies a specific choice of A_2 . Given this the mass of firms in location 2 is $\hat{\mu}_2(\varphi)$:

$$\hat{\mu}_2(\varphi) = \begin{cases} z_l \text{ if } \varphi < z_l \\ \frac{1}{1-\xi} (z_h - z_l) \text{ if } \varphi \in [z_l, z_h] \\ 1 - z_h \text{ if } \varphi > z_h \end{cases} \quad (40)$$

Note that firms in location 2 with $\varphi < z_l$ exit and there is a measure z_l of them. Firms with $\varphi > z_h$ move to 1, and there is a measure $1 - z_h$ of them. Firms in the middle group $\varphi \in [z_l, z_h]$ remain in 2 forever subject to surviving the death shock ξ .

Let m denote entry in location 1. The mass of firms in location 1 is:

$$\hat{\mu}_1(\varphi) = \begin{cases} m z_l \text{ if } \varphi < z_l \\ \frac{m}{1-\xi} (z_h - z_l) \text{ if } \varphi \in [z_l, z_h] \\ \frac{(\xi+m)}{1-\xi} (1 - z_h) \text{ if } \varphi > z_h \end{cases} \quad (41)$$

Firms in the first group exit immediately. Firms in the middle group stay in 1 forever. Firms with $\varphi > z_h$ come from 2 sources: 1. firms who entered in 1 and stayed there forever subject to death shock $m(1 - z_h) / (1 - \xi)$ plus firms who entered in location 2 last period, survived the shock and moved to 1 where they remain forever: $\xi(1 - z_h) / (1 - \xi)$. Q.E.D.

Finally, we have the following result:

Proposition 2.4 *There is at most one value of m such that the relative demand for land equals the relative supply of land. Under conditions on the parameters, m is shown to exist.*

Proof:

Given the equilibrium distributions, we can solve for equilibrium value for entry, denoted

by m . Note that given the assumptions the demand for labor is:

$$l_j(\varphi) = \bar{l} + \left(\frac{\alpha}{w}\right)^\eta \varphi^{\frac{1}{1-\alpha-\gamma}} r_j^{\frac{1-\alpha}{1-\alpha-\gamma}}. \quad (42)$$

The equilibrium value of m is such that it solves the relative land equilibrium condition which can be written as

$$\int l_1(\varphi) \hat{\mu}_1(d\varphi) = \frac{A_2 r_1}{A_1 r_2} \int l_2(\varphi) \hat{\mu}_2(d\varphi) \quad (43)$$

where the right hand side does not depend on m . The left-hand side depends linearly in m through the mass $\hat{\mu}_1(\varphi)$ in an increasing way. This means that if m exists it is unique.

For $m \rightarrow \infty$ the left hand side of (43) goes to infinity. For $m \rightarrow 0$ the left hand side is strictly positive. To show that it is less than the right hand side A_1 must be sufficiently small. Since the rest of the equilibrium is independent of A_1 one can always choose A_1 small enough in order to guarantee existence. Thus, there exists a unique value of m . Q.E.D.

In what follows we present the equilibrium of the model in a numerical example.

Result 2.1 *Consider the following parameter values: $\beta = 0.5$, $\alpha = 0.65$, $\theta = 0.35$, $\xi = 0.9$, $\bar{l} = 0.01$, $\gamma = 0.01$, $\eta = 2.94$, $w = 1$, $\Delta = 0.0367$, $A_1 = 0.5$, $A_2 = 1.0$, $c_e = 0.1$, $c_r = 0.01$, $\delta = 1$. Then, the unique equilibrium of the model is characterized by the following: $\varphi_l = 0.64$, $\varphi_h = 0.69$, $r_1 = 37.18$, $r_2 = 34.98$, $m = 0.21$.*

The analysis of this section shows that there exists a unique (up to scale) equilibrium with entry in both locations. Our analysis in the previous section reinforces the notion that equilibria with entry in both locations are often locally unique.

Chapter 3

Transportation Technologies, Agglomeration, and the Structure of Cities

3.1 Introduction

Public provision of transportation networks has long been used as a driver of economic development within urban areas. There is no doubt that the provision of roads and transit contribute to economic growth and development by increasing access to land, decreasing commuting costs, and in general, improving access for all economic agents in a local economy. It is also an uncontroversial paradigm that these transportation networks are provided publicly, given that transportation networks include characteristics of both public goods and natural monopolies. However, there is disagreement on the ideal mix of transportation networks in terms of the provision of transit versus automobile infrastructure. During the later half of the twentieth century in the United States, transportation investment focused on improving the automobile infrastructure over transit, and spending on transportation remains heavily skewed toward roads and highways at every level of government. Nonethe-

less, concerns about energy costs, the environment, increased traffic congestion, and loss of open space, have enlivened the debate in recent years. The full range of costs and benefits of different transportation technologies is far reaching and includes access, efficiency, and environmental concerns, along with the costs of construction and maintenance of infrastructure and service. This paper focuses on a different aspect of transportation provision by exploring how different transportation technologies affect the overall spatial structure of cities, as well as commuting patterns. The increased understanding of the spatial structure of cities provided here is well suited to enhance analysis of new and popular urban policies, including congestion pricing, transit oriented development, targeted development subsidies, and general provision of transportation infrastructure.

To fully explain the structure of cities, we must also consider the effects of agglomeration economies, which occur when firms gain production benefits from proximity to other firms in the form of decreased transaction costs or locational economies of scale. Without the presence of agglomeration economies, it is impossible to explain the observed extent of economic clustering. Previous literature has sought to describe the underlying structure of cities in the presence of transportation costs and agglomeration externalities. The key theme of this literature has been the conflict between these two contraction forces and the scarcity of land, which is the main dispersion force. Researchers have applied these forces to characterize the geographical structure of cities in terms of size, land uses, densities, rent, and wages. A starting point for this research is the classic work by Von Thünen (1826), Mills (1967), and others who pioneered urban economics, and developed the circular city model with a central business district. This general line of study was then followed by others, including Solow (1972), who added congestion costs, and Dixit (1973), who allowed the size of the central business district to affect the productivity of firms in the economy through returns to scale.

Since the publication of these papers, two important developments have taken place. The first is the simple observation that the importance of the central business district has declined and therefore the single business district model is no longer rich enough to describe the true structure of cities. The second is the emergence of a large literature

which helps to better understand the agglomeration economies first proposed by Marshall (1890) and further dissected by Jacobs (1969). Important work on the scale, structure, and determinants of agglomeration externalities includes, among others, Rosenthal and Strange (2001), Duranton and Puga (2001), Henderson et al. (1995), Krugman (1991), and Fujita and Thisse (2002).

Other recent literature has focused on developing full general equilibrium models of urban structure. Anas and Kim (1996) present a linear city model which places no restrictions on the location of firms and consumers, and find that multiple commercial centers can emerge if there are significant enough scale economies. I choose to adopt the structure of Lucas and Rossi-Hansberg (2002), henceforth LRH, who present a full general equilibrium model in a circular city setting which includes transport costs and production externalities, and also find the emergence of multiple centers.

This paper extends the LRH model so that empirical evidence can be used to illustrate and test the important characteristics of the model. In particular, by allowing for a very local complementarity of land uses, the current model is able to better explain the observed extent of local mixing of commercial and residential land use. In addition, the current work focuses on the effects of different transportation technologies on the structure of the city in terms of spatial distribution of firms and workers in the presence of agglomeration externalities. In particular, I include a congestion cost in the general equilibrium model, which adds an external cost to the commuting decisions of workers, in addition to the distance cost. Specifying the model in this manner allows for different transportation technologies to behave differently in areas of high or low congestion.

The rest of the paper is organized as follows. Section 2 introduces data from three cities to exhibit urban spatial structure in terms of commercial and residential densities, wages, and commuting times, and to illustrate the trade-offs faced by businesses and individuals in urban location decisions. Sections 3 and 4 develop the model, define equilibrium conditions, and describe a computational algorithm to solve for equilibrium. Sections 5 and 6 outline the estimation procedure and use the estimated model to gain insight into the effects of

different transportation infrastructure provision on the structure of cities. Overall, the estimates illustrate the importance of congestion and local complementarity of uses, and the quantitative results explain the important characteristics of the city including population densities, employment densities, and commuting times. Finally, section 7 offers policy experiments which show that reducing congestion costs leads to higher density central business districts and exurban areas. In addition, residential density becomes more concentrated in inner ring suburbs.

3.2 The Structure of Cities

Before introducing the theoretical model, it is important to provide an empirical description of the structure of cities. It has been well established that the structures of cities result from competing forces which act on the spatial distribution of economic activities, and can be categorized as contraction forces and expansion forces. Contraction forces lead to higher density, and include agglomeration externalities, where firms prefer proximity to other firms, and commuting costs, where individuals prefer to live closer to where they work. Expansion forces are primarily driven by scarcity of land, and the demand of both firms and individuals for more land and, in turn, cheaper rents.

Given this framework, the characteristics of interest for this research primarily lie in the spatial distribution of residential density, commercial density, wages paid, and commute times. In particular, we are interested in how these quantities change in relation to the distance from the center of the city. For illustrative purposes, data are presented for three metropolitan areas, Columbus, Ohio, Philadelphia, and Houston. These cities differ in both size and transportation networks as illustrated in Table 3.1, which allows for a point of reference in comparison of the cities.

	Columbus	Houston	Philadelphia
Total Employment (MSA)	845,815	2,100,380	2,559,383
Percent Transit Commuting (MSA)	0.97	3.3	9.2

Table 3.1: Employment and commuting mode characteristics of selected cities

In order to understand the spatial structure of cities in terms of densities and wages, data was collected from the the 2000 Census Transportation Planning Package available through the Bureau of Transportation Statistics.¹ The data set is a subset of the Decennial Census Sample Data, but is organized geographically both by residential and employment location of workers, to allow for analysis of employment, residential, and commuting patterns in a spatial context.

I processed the data using GIS software to calculate densities and wages as a function of the distance from the center of the central business district, which is fairly well defined for all three cities. The top panel of Figure 3.1 shows the residential density as a function of radius in miles. For all three cities, residential densities decline substantially from the center of the city outward. However, Philadelphia is unique, in that it maintains a much higher residential density near the city center. The higher transit provision in Philadelphia could explain part of this difference. Houston and Columbus follow similar patterns (taking the overall size difference of the cities into account), with little residential density in the central business district, followed by higher density and then gradually declining density.

The middle panel of Figure 3.1 shows the employment density gradient. For all three cities, there is considerably more variance in employment densities compared to residential density, and all three cities exhibit high density central business districts with rapidly declining employment moving outward. In addition, Philadelphia retains a higher density central business district over a larger area, while Houston has a smaller central business district but density declines less rapidly, even exhibiting a second business district away from the city center.

Finally, the bottom panel of Figure 3.1 shows the wages paid by employers as a function of radius. Wages also decline away from the city center. The key observation here is that it appears that firms pay a premium to be located in high density areas. These wage gradients provide evidence of agglomeration economies, but given heterogeneity in both firms and workers, the raw wage data should be interpreted with a measure of skepticism.

¹The data set is available at <http://www.transtats.bts.gov>.

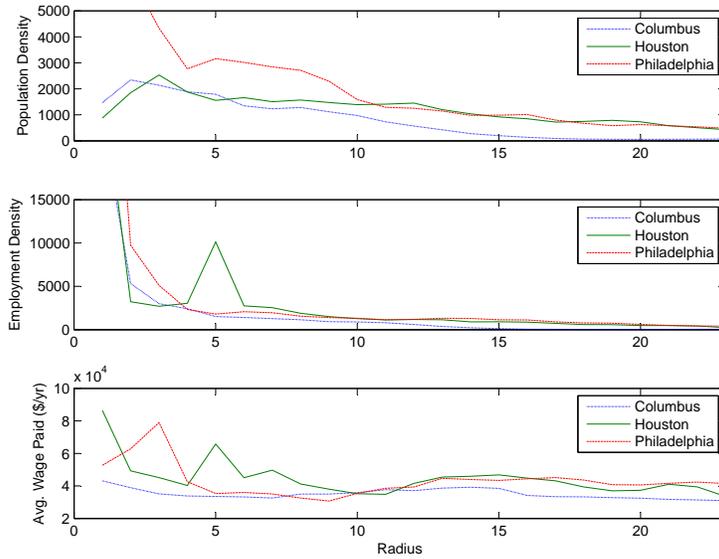


Figure 3.1: Residential density, employment density, and wages paid as a function of radius from the city center for selected cities

In addition to the density and wage gradients, it is useful to look at the data in two dimensions to gain a sense of the overall spatial distribution of economic activity. Figure 3.2 maps the population densities, employment densities, and average wages paid, along with a measure of commuting costs, average commute times, for individual census tracts in Columbus. Note that because wages and commuting times are not spatial measures, variances are heavily influenced by employment and population respectively in each tract, so they should be interpreted carefully. Nonetheless, commute times appear to show a clear increasing pattern moving away from the city center.

Beyond what was already exhibited by the gradient plots, the important takeaway from these maps is that Columbus exhibits a strong radial (and nearly circular) pattern spatially for all measures of interest. The theoretical model assumes circular symmetry, so this is an important feature of the data. Admittedly, all cities do not display such a circular pattern, but it is encouraging that Columbus, a city located in mostly flat farmland, away from major bodies of water, exhibits a fairly circular pattern. For this reason, data from

Columbus are used for the remainder of the empirical analysis.

Overall, the data suggest, first, that despite presumably significantly higher rents in the city center, firms benefit from higher agglomeration externalities in denser areas. In addition, it appears that workers will sacrifice land consumption for shorter commute times. These effects are exhibited in both densities and wages for all three cities. Also, commercial densities display larger variances than residential densities. In general, the goal of the current research is to explain the characteristics of these data, by modeling and estimating the trade-offs and equilibrium effects of agglomeration and commuting costs, while paying special attention to the effects of commuting congestion, to better understand how transportation technologies play a role in the spatial structure of cities.

3.3 The Model

The model assumes a circular city and considers only symmetric allocations. Beyond these assumptions, no restrictions are placed on the location of businesses or residents. The model draws heavily on the work of Lucas and Rossi-Hansberg (2002) and Rossi-Hansberg (2004). In these papers, the authors prove the existence of equilibrium, and discuss the optimal allocation in an economy with transportation costs and production externalities. These papers provide a general well-formed theory of the forces affecting job and residential distributions within an urban area. The key departure of the current research is that congestion is added to the transportation cost, as opposed to a simple distance cost. Note that this adds a second externality in the model, in that individual workers' commuting decisions now place an external cost on the economy. By more precisely modeling the transportation cost in the economy, we are able to consider transportation technologies that have different distance and congestion costs, and therefore different effects on the spatial structure of the city in equilibrium.

Additionally, the model has been modified to allow for more mixing of commercial and residential uses. The LRH model places sharp restrictions on land use, and allows for

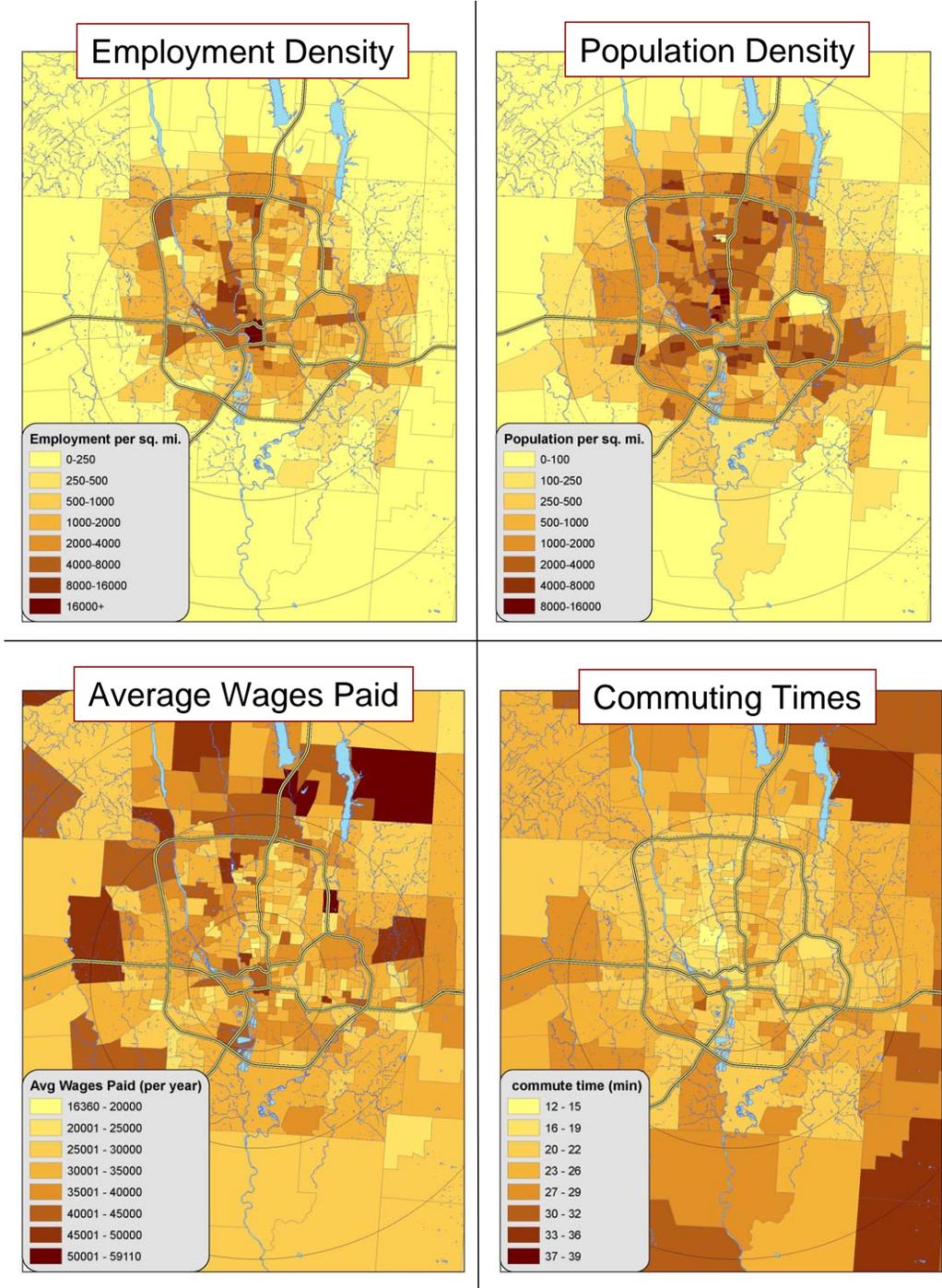


Figure 3.2: Spatial distribution of employment, population, wages paid, and commuting times for Columbus, Ohio

mixing only under precise conditions. Empirical observation suggests, however, that there is a great deal of mixing of land uses at every geographic scale. Furthermore, observed land use patterns exhibit more gradual transitions than those produced by LRH. To address this disconnect between the theory and data, a local complementarity of uses is added to the model. These changes, along with all the basic assumptions of the model, are discussed here.

Assumption 3.1 *The city is circular and allocations are symmetric. Additionally, commuting is always radial, and the size of the city is fixed at radius, S .*

The assumption of symmetric equilibria implies that all allocations can be written as a function of r , defined as the distance from the center of the city, and not the angle, ϕ (using standard circular coordinates). The assumption of circular symmetry is strong, however, these symmetric equilibria are feasible in a general sense. Still, symmetric equilibria are not necessarily stable in the presence of asymmetric shocks. Nonetheless, in the interest of tractability and generalizability, the circular assumption is useful.

The model uses the following notation. $\theta(r)$ is the fraction of land used for production at location r and $1 - \theta(r)$ is the fraction of land used for residential. $n(r)$ is the employment density at r defined as employment per unit of production land, and $N(r)$ is the residential density defined as the number of workers housed per unit of residential land. Additionally, land is owned by an absentee landlord. Given these definitions, we can now describe the preferences of consumers and the technology of producers.

Assumption 3.2 *Workers supply one unit of labor inelastically and have increasing preferences over general consumption, $c(r)$, and land, $l(r)$.*

Worker preferences are modeled using a Cobb-Douglas form given by, $U(c(r), l(r)) = c(r)^\beta l(r)^{1-\beta}$. Given this function form, β is interpreted as a consumption share parameter.

Assumption 3.3 *Firms produce an outside good through a production function that is increasing in land and labor, and firms are subject to an agglomeration externality.*

The production function is modeled as constant returns and Cobb-Douglas and the production per unit land is given by,

$$x(r) = g(z(r))f(n(r)),$$

where g is the production externality, given by

$$g(z(r)) = z(r)^\gamma$$

and f describes the land and labor technology, given by

$$f(n(r)) = An(r)^\alpha.$$

In the above form, γ determines the scale of the externality, while α is the ratio of the share of labor and land. Also, note that the constant returns specification allows for a single production input variable defined as labor per unit land, as opposed to including land and labor separately.

Assumption 3.4 *The externality depends on a proximity measure, $z(r)$, of a firm located at $(r, 0)$ to other firms in the economy located at (s, ϕ) . This measure is assumed to be increasing in employment of other firms, and attenuates with distance from other firms.*

The functional form of the externality measure is given by,

$$z(r) = \delta \int_0^S \int_0^{2\pi} s\theta(s, \phi)n(s, \phi)e^{-\delta d(r, s, \phi)} d\phi ds, \quad (1)$$

where δ determines the rate of exponential decay of the externality with distance, and d is the distance between two firms given by

$$d(r, s, \phi) = [r^2 - 2\cos(\phi)rs + s^2]^{1/2}.$$

Note that this externality is only a function of the radius, r , and not the angle ϕ , given the

symmetry assumption.

The next component of the economy is a commuting cost.

Assumption 3.5 *Workers pay a cost to travel to work which is subtracted from their wage. This cost is increasing in both distance traveled and in congestion.*

At this point, the model diverges from the LRH model. The commuting cost is paid by workers, meaning that each worker always delivers one unit of labor to the location of the firm, but experiences a loss of wages equal to the commuting cost.² This means that the wage that enters into the workers budget constraint is the wage she earns minus commuting costs. Furthermore, the current model assumes that this commuting cost is a function of both distance and congestion. The form of the commuting cost is similar to that proposed by Dixit (1973). The total commuting cost between a location s and location r will be denoted $T(s, r)$ and can be written using the following parameterization for the marginal cost of commuting through location r as,

$$T'(r) = \tau + \kappa m(r)$$

where $m(r)$ is the congestion, defined as commuters divided by circumference at each radius, r . In other words, holding the total mass of commuters constant, congestion will increase closer to the center of the city. The parameters τ and κ can be interpreted as the distance and congestion costs respectively. Using this specification, we can consider transportation technologies (or mixes of technologies) that exhibit different costs in the presence of different congestion levels. For example, in the context of intraurban transportation, one could conjecture that transit systems provide relatively low costs in congested areas, while automobiles provide relatively low costs in uncongested areas. Note, that these costs do

²In LRH, the commuting cost is modeled as a loss of labor to firms. This difference in specification does not change the incidence of transportation cost in equilibrium, only the interpretation of who pays the cost. The LRH specification has advantages in theoretical analysis, while the current specification enhances empirical analysis, by allowing the researcher to relate model characteristics more directly to data such as residential location, job location, and commute times.

not include the public costs of construction, maintenance, or operation of transportation services, but merely the costs faced by individual commuters.

Given the dependence of transportation costs on congestion, it is important to further investigate the congestion function, $m(r)$. Consider that the total number of commuters at a location, r , is the sum of all jobs from 0 to r less the number of residents from 0 to r . The congestion is then the number of commuters through a location divided by circumference at location r . Taking all of this together, the congestion function is the following.

$$m(r) = \frac{1}{2\pi r} \int_0^r 2\pi r' [n(r')\theta(r') - N(r')(1 - \theta(r'))] dr' \quad (2)$$

This congestion function can be positive or negative, where positive values represent commuting toward the center of the city, while negative values represent commuting away from the center. A value of zero represents zero commuting, and also represents a radius containing equal masses of jobs and residents. Note that $m(r)$ is determined directly from $n(r)$, $N(r)$, and $\theta(r)$.

One could assume that land is allocated to the highest bidder.³ (i.e. if the commercial bid rent is higher than residential bid rent, then all land is used for commercial purposes and visa versa.) This sharp restriction is hard to justify empirically, given that, in reality, one observes a large amount of mixing of uses at all geographic scales. This points to some complementarity of uses at a very local neighborhood level. An obvious example is in the retail sector where proximity to customers is much more important than proximity to other businesses. Likewise, workers may have preferences to be located near businesses for reasons other than commuting. While I do not specifically study the determinants of this mixing, in order to successfully compare the model to the data, I include a local complementarity of uses in the specification of land supply.

Assumption 3.6 *The landlord will seek to maximize rent revenue per unit land at every location, but is subject to a transformation function for land services describing the comple-*

³This is the the assumption made by LRH

mentarity of land uses.

A constant returns constant elasticity function describes this transformation of land services. The landlord's maximization problem for commercial and residential allocation, $\theta_C(r)$ and $\theta_R(r)$ respectively, at each location r , for commercial and residential rents, $q(r)$ and $Q(r)$ respectively, is then,

$$\max_{\theta_R(r), \theta_C(r)} Q(r)\theta_R(r) + q(r)\theta_C(r) \quad s.t. \quad \left[\eta\theta_R(r)^{\frac{\rho-1}{\rho}} + (1-\eta)\theta_C(r)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} = 1, \quad (3)$$

where $\eta \in (0, 1)$ is a share parameter, and $\rho \in [-\infty, 0]$ is the elasticity between land uses, which is negative to reflect that the uses are complements.⁴

The equilibrium definition invokes some of the usual conditions. Firms will maximize profits at every location, r , given wages, $w(r)$, commercial rents, $q(r)$, and the productivity, $z(r)$. Workers will maximize utility at every location, given net wages, $w(r)$, and residential rents, $Q(r)$. The landlord will maximize rent at every location given commercial rents, $q(r)$, and residential rents $Q(r)$. In addition, the market for land must clear at every location, which is implied by the land transformation specification.

However, several other conditions are needed to describe equilibrium in a spatial context, given the mobility of both firms and workers.

Assumption 3.7 *The city exists in a larger economy and both firms and workers are free to enter or exit.*

The above assumption implies a zero profit restriction on firms in equilibrium. For workers, the assumption suggests that there is a reservation utility, \bar{u} , which must be obtained with equality at every location, r , in the city in equilibrium. Since, workers and firms will achieve identical utility and profits, respectively, at every location, then they will have no

⁴I show in the computational section that this specification produces a continuous land use function which simplifies equilibrium computation.

incentive to relocate in equilibrium, so no additional condition is necessary to ensure that firms or workers cannot be made better off by relocating within the city.

However, to ensure that workers have no incentive to commute to a different location, equilibrium requires a restriction on the wage gradient through commuting costs. The necessary condition in equilibrium is that the difference in wages paid between two locations must be equal to the total commuting cost of traveling between the two locations. Given the functional form of the marginal commuting cost, this condition can be written as,

$$w(r) - w(s) \leq \int_s^r (\tau + \kappa m(r')) dr', \forall r, s \in [0, S]. \quad (4)$$

It is straight forward that workers will only travel toward higher wages compared to the wages paid where they live. To gain further intuition into this condition, consider the situation where the difference in wages is greater than the commuting cost between locations. If this were the case, then workers would all desire to work at the high wage location. Likewise, if the difference in wages were less than commuting costs between locations, then all workers would desire to work at the low wage location. Given that the land supply function requires some employment at all locations, this condition must hold everywhere.⁵ This condition allows us to use a single notation for wages, $w(r)$, given that the wages paid must equal the net wages (wages paid minus commuting costs) at every location, despite the distinction.

Finally, to close the model we require a labor market clearing condition, which states that all workers must be housed within the city. An equivalent condition is that commuting is zero at the edge of the city, which can be formally written as, $m(S) = 0$. The underlying assumption here is that commuting costs from other cities are significantly high to prevent such activity. All of the pieces are now in place to formally define equilibrium.

⁵LRH place more complex restrictions on this wage path given that their model has a non-continuous land use function, $\theta(r)$, allowing for regions where there is no commuting. While the current model allows commuting to be zero at a single location, the no-commuting region cannot maintain over any distance, given the additional restrictions of the land supply function.

Definition 3.1

Equilibrium is defined as a set of allocation functions $\{z(r), \theta(r), n(r), N(r), m(r)\}$ along with a set of price functions $\{w(r), q(r), Q(r)\}$ defined on $[0, S]$, such that for all r ,

- i . Firms choose $n(r)$ to maximize profits at all locations, given $z(r)$, $w(r)$, and $q(r)$*
- ii . Workers choose $N(r)$ to maximize utility at all locations, given $w(r)$ and $Q(r)$, subject to their budget constraint.*
- iii . Landlords maximize rents given, $q(r)$ and $Q(r)$, subject to (3)*
- iv . $m(r)$, $n(r)$, $N(r)$, and $\theta(r)$ satisfy (2), and $m(0) = 0$*
- v . Firms achieve zero profits and workers achieve a reservation utility, \bar{u} at every location, r .*
- vi . $w(r)$ must satisfy (4) and thhe labor market clears, i.e. $m(S) = 0$.*
- vii . z , θ , and n satisfy (1)*

3.4 Computation of Equilibrium

This section outlines the computational methods used to calculate equilibrium given the functional form choices introduced above. The algorithm uses a shooting algorithm to find a wage path consistent with the labor market clearing condition, which is nested inside a fixed-point algorithm, which ensures that the agglomeration externality is consistent with the allocation of employment. The model is able to produce a wide range of employment and residential distributions and commuting patterns.

The equilibrium solution starts by deriving the equilibrium choices for firms, workers, and landlords. Solving the profit maximization problem of the firm, the indirect labor choice per unit land as a function of wages and the externality is,

$$n(r) = \left(\frac{\alpha A z(r)^\gamma}{w(r)} \right)^{\frac{1}{1-\alpha}},$$

where $w(r)$ is the wage paid by a firm at location r . Using the indirect labor function, along with the zero profit assumption, we can solve the equilibrium commercial rent, $\hat{q}(w(r), z(r))$, as a function of productivity z , and wage w .

$$q(r) = (1 - \alpha) \left(\frac{\alpha}{w(r)} \right)^{\alpha/(1-\alpha)} A^{1/(1-\alpha)} z(r)^{\gamma/(1-\alpha)}.$$

In a similar fashion, we can solve for an individual's indirect labor density choice and equilibrium residential rent. Individuals will maximize utility subject to their wage, $w(r)$. Recall that wage, $w(r)$, takes a different meaning for individuals, in the sense that $w(r)$ is the net wage defined as wage paid less commuting costs. The city's existence in a larger economy suggests that individuals have a reservation utility, \bar{u} , which gives the following condition.

$$c(r)^\beta l(r)^{1-\beta} = \bar{u}.$$

With this condition, solving for the indirect residential density gives,

$$N(r) = \beta^{\beta/(1-\beta)} \bar{u}^{-1/(1-\beta)} w(r)^{\beta/(1-\beta)},$$

and the equilibrium residential rent,

$$Q(r) = \left(\frac{w(r)}{\bar{u}} \right)^{1/(1-\beta)} \beta^{\beta/(1-\beta)} (1 - \beta).$$

The rent maximization problem of the landlord, along with the transformation of land services gives the following land supply function which is based on the ratio of commercial and residential rents in a location.

$$\theta(r) = \frac{1}{1 + \left(\frac{q(r)}{Q(r)} \frac{\eta}{1-\eta} \right)^\rho}$$

This is a very flexible form and allows for complete segregation of uses (corresponding to $\rho = -\infty$) or complete mixing of uses (corresponding to $\rho = 0$). In addition, there

can be asymmetry in the complementarity for highly residential areas compared to highly commercial areas. An additional benefit of this form is that it produces a continuous land use function, $\theta(r)$, which is useful in computation of equilibria for the model. This result produces a functional form for land use similar to that used by Wheaton (2004).

Finally, the transportation costs, in concert with free mobility of labor, imply that wages must adhere to the following spatial restriction.

$$w(r) - w(s) = \int_s^r \tau + \kappa m(r') dr', \forall r, s \in [0, S]$$

Notice that for a given congestion, $m(r)$, and an initial wage at the center of the city, $w(0)$, the entire wage path can be calculated. This wage path can be increasing or decreasing depending on the direction of commuting.

We can now describe the equilibrium solution algorithm. The solution algorithm can be thought of as an inner loop which searches for an initial wage, $w(0)$, leading to a wage path and allocations consistent with equilibrium (conditions $i - vi$) for a given productivity, $z(r)$, and an outer loop which consists of an iterative fixed point algorithm to find a productivity function consistent with the externality specification, (condition vii).

The algorithm starts by guessing an initial productivity function, $z(r)$. In the inner loop of the algorithm, this productivity function is taken as given. The next step is to guess an initial wage, $w(0)$. With the initial wage, we can construct the entire wage path, and hence, all of the allocations of the economy. Given that $w(0)$ and $z(0)$ are known, we can calculate $\hat{n}(0)$ and $\hat{N}(0)$, and the congestion $m(0)$. Knowing the congestion allows for the calculation of the wage at the next location $r = 0 + \epsilon$, and determines the direction of commuting. We can then calculate the allocations and congestion at $r = 0 + \epsilon$. The algorithm continues to move outward from the city center, until it reaches the edge of the city $r = S$. At this point, the algorithm checks that the labor market clears, or that $m(S) = 0$. Given the current functional specification, I have confirmed computationally that $m(S)$ is a decreasing

continuous function of $w(0)$.⁶ Therefore, from this point, any minimization routine can find the initial wage such that the labor market clears.

The process of constructing a wage path is demonstrated in Figure 3.3, which shows an example of wage path plotted with congestion. Note that the slope of the wage path is correlated with the congestion. Also, although not shown graphically, the congestion is dependent on the the relative levels of commercial and residential density and the land use function. This example exhibits a particularly complex wage path and shows commuting patterns changing within the city. A negative congestion (or positive slope on the wage path) represents outward commuting. This example illustrates the flexibility of the model. In the estimated model, commuting is always inward, represented by a decreasing wage path.

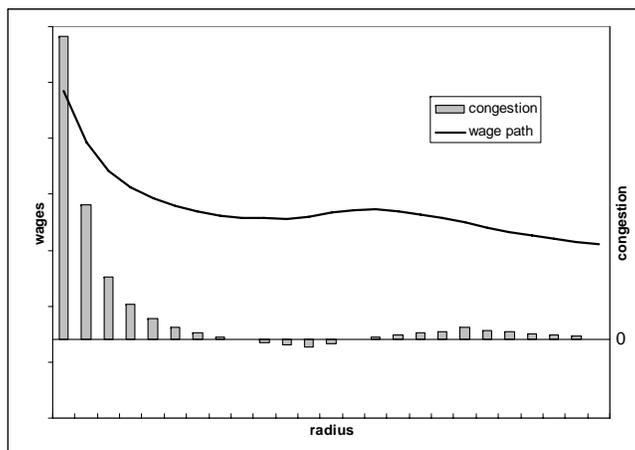


Figure 3.3: Example of wage path construction for given production externality

The outer loop of the algorithm then uses the commercial density function, $n(r)$, along with the land use function $\theta(r)$, to calculate the theoretical productivity, $z(r)$. The productivity function is updated, and the routine repeats the inner loop. It continues this process until $z(r)$, converges to a fixed point. At this point the algorithm has found allocations and

⁶Note that the continuity of this function arises from the land supply specification. This reduces the complexity of the solution algorithm compared to LRH, where the correspondence between $m(S)$ and $w(0)$ was decreasing but not continuous, given that the land use function was discontinuous.

prices consistent with conditions *i – vii* of the equilibrium definition.

3.5 Estimation

The above solution algorithm implicitly defines a non-linear mapping from the parameter vector of the structural model, from here on denoted Θ , to a distribution of equilibrium outcomes. This allows us to match computational outcomes to observed outcomes in the data. In order to estimate the parameter vector, Θ , it is convenient to match select aggregate moments using a method of moments estimator, similar to that suggested by Hansen (1982).⁷

Denote the vector of sample moments as m_N , and denote with $m_S(\Theta)$ the vector of equivalent computational moments calculated through the solution algorithm. Using this notation, a vector of orthogonality conditions is defined as

$$g_{N,S}(\Theta) = m_N - m_S(\Theta).$$

Θ can then be estimated using the following consistent estimator which minimizes the weighted distance between the sample moments and the computational moments.

$$\hat{\Theta}_N = \arg \min_{\Theta} g_{S,N}(\Theta)' A_N g_{S,N}(\Theta)$$

for some positive semi-definite matrix A_N which converges in probability to A_0 . The asymptotic distribution of the estimator is then given by,

$$N^{1/2} (\hat{\Theta}_N - \Theta_0) \xrightarrow{d} N(0, (\tilde{A}_0 D_0)^{-1} \tilde{A}_0 V \tilde{A}_0' (\tilde{A}_0 D_0)^{-1'})$$

where $\tilde{A}_0 = D_0' A_0$, $D_0 = E[\partial m(\Theta) / \partial \Theta_0]$ and V is asymptotic covariance matrix of the vector of sample moments. In addition, the most efficient estimator is obtained by setting

⁷A similar application, with further discussion of this estimation method can be found Brinkman, Coen-Pirani, and Sieg (2010)

$A_N = V_N^{-1}$. Then the asymptotic distribution is the following:

$$N^{1/2} (\hat{\Theta}_N - \Theta_0) \xrightarrow{d} N(0, (D_0' V^{-1} D_0)^{-1})$$

I estimate the model using data from Columbus, described earlier, given that the city has a desirable geography in terms of the circular assumption of the model. In addition, the transportation infrastructure is radial and nearly completely automobile oriented, providing a good baseline, and avoiding the complication of different provision of transportation infrastructure.

The moments used in the estimation are effectively the population and employment densities, along with commuting times. These moments are separated into four locations, to identify the spatial relationship of densities and wages. The four locations used, based on distance from the center of the city, are 0 to 5 miles, 5 to 9 miles, 9 to 12, and 12 to 23 miles. These distances are chosen to give approximately even employment and population in each area, but also because they could reasonably be interpreted as the urban, inner-suburban, suburban, and exurban regions within the metropolitan area. This lends some intuition to the interpretation of results.

Several difficulties arise in calculating the moments and in the estimation due to the nature of the data available. The first difficulty is that, in reality, firms and workers are not homogeneous, and therefore applying the observed wage data directly would certainly result in biased estimates. Furthermore, there is no obvious clean way to control for this heterogeneity that is independent of spatial relationships. For this reason, I use commuting times as a proxy for commuting costs. However, in order to use commuting times as reported in the data, one additional assumption is needed to precisely identify commuting patterns in the theoretical model.

The equilibrium solution to this point pins down aggregate commuting through a given location, but does not identify the residential origin of the commuters. To provide intuition, denote the mass of commuters moving through a location, r , as $M(r)$. When these com-

muters arrive at a given location r , they are indifferent between filling a job at that location or commuting on, given that they are exactly compensated for additional commuting with additional wages. Therefore, the model does not specify which commuters fill jobs at any given location.

This means that the average commuting cost is not defined by residential location. Empirically this creates a problem given that commute times are reported by residential location. To solve this problem, we need a rule which defines how jobs are filled at any given location. Given that commuters are indifferent, an obvious assumption is that commuters fill jobs with equal probability regardless of residential origin.

Assumption 3.8 *At any given job location, commuters are equally likely to fill available jobs, regardless of residential origin.*

This implies that the proportion of jobs filled at any given location, r' , by commuters originating from r is equal to the ratio of the mass of commuters originating from r to the total mass of commuters at r' . If we denote the mass of commuters originating at r who pass through r' as $M_r(r, r')$, then the following differential equation defines the change in $M_r(r, r')$, given $M(r')$.

$$\frac{\partial M_r(r, r')}{\partial r'} = \frac{M_r(r, r')}{M(r')} (2\pi r' n(r') (\theta(r')))$$

This differential equation, along with initial conditions at a small epsilon inside the edge of the city,

$$M(S - \epsilon) = M_r(S - \epsilon, S - \epsilon) = 2\pi S (N(S)(1 - \theta(S)) - n(S)\theta(s)),$$

allows for computation (numerically) of the commuting patterns for all workers by residential location. Given the commuting patterns by residential location, the average commuting

cost by residential location is given by,

$$\frac{1}{2\pi r N(r)(1 - \theta(r))} \int_0^r M(r, r')(\tau + \kappa m(r')) dr'$$

The above discussion shows how commuting costs in dollars can be calculated from the theoretical model. To compare this to the data, commute times must be converted into a monetary cost. I convert commuting times to lost wages by assuming that commuting costs are equal to half of wages multiplied by commuting time. For example, if a person who makes \$10 per hour commutes for one hour, the commuting cost is \$5. This is consistent with estimates from academic literature and is also standard in cost-benefit analysis of transportation projects.⁸ Finally, one additional adjustment needs to be made to the commuting data. While the model produces zero commuting costs for some locations, in the data, commute times are never zero. This implies that there is some fixed commuting cost which is present for all workers. In other words, if one were to regress commute times on distance from the center of the city, there is an obviously significant intercept term. This can be modeled with an additional parameter, c_f , interpreted as a fixed cost. It is straight forward that this parameter is equivalent to a shift in wages for all workers, but it is useful to apply the cost directly to commuting to match the model and data.

An additional challenge lies in the use of aggregate spatial data in the method of moments estimation. A common way to deal with aggregate data is to weight the observations in both the calculation of sample moments, m_n , and the covariance matrix, V , by the population of the observation. However, for this application, the populations are not consistent for different variables (i.e. population densities and employment densities are normalized differently). The consistent weight for all variables is the area of a particular tract. Moments must be constructed that are normalized by area so that when we aggregate these moments, weighting by area, the aggregate moments are correct. Population and employment densities are already in this form.

⁸See Tse and Chan (2003) and Small (1983) for estimates in the literature.

It is straight forward to show that the area weighted average of density will produce the correct aggregate density. For the commuting costs, I use a commuting cost density, (i.e. total commuting costs per square mile) so that the area weighted averages will produce the correct aggregates. The weighted covariance matrix is also calculated using these area weights. The final moments used are then population density, employment density, and commuting cost density, all interacted with location dummy variables to capture the spatial relationship of the moments.

Finally, the radius of the city is set at 23 miles. Ideally, we would like to set the edge of the city at a location where the total employment is equal to the total number of workers. However, in reality, the market does not clear even well into sparsely populated areas. Analysis of the entirety of rural areas is not interesting in the context of urban spatial structure, so a judgment was made to stop the city at a radius of 23 miles, where densities are very low. Note, that this requires that we explicitly allow 56,157 unhoused workers, in order to match the data in the estimation.

3.6 Quantitative Results

The parameter estimates are shown in Table 3.2. All of the parameters are significant. Several parameter estimates should be highlighted. $1 - \beta = .014$, can be interpreted as the share of income spent on land by individuals. This is less than previous estimates in the literature, albeit on a similar scale.⁹ This difference may be explained by the fact that land is cheaper than average in Columbus. The production share of land, α is consistent with the range of previous estimates.¹⁰

The commuting cost parameters also have intuitive real world interpretation. The distance cost parameter, $\tau = 94.9$, can be interpreted as the cost per year per mile commute in a completely uncongested area. For perspective, if we assume 260 work days per year,

⁹See Roback (1982), for a discussion of land share of utility

¹⁰Estimates of land share of production vary in the literature. For further information see, Caselli and Coleman (2001), Deckle and Eaton (1993), and Adsera (2000).

Parameter	Description	Estimate	Standard Error
β	utility consumption share	0.986	6.17e-6
α	land and labor technology	0.790	8.43e-4
γ	externality scale	0.214	7.92e-4
δ	externality attenuation	0.580	1.03e-4
τ	distance cost	94.9	3.93e-2
κ	congestion cost	0.0117	2.77e-5
A	productivity scale	25187	8.32
u	reservation utility	26827	1.96
η	land services ratio	0.136	2.18e-3
ρ	land services elasticity	-0.630	2.14e-3
c_f	fixed commuting cost	1,430	5.17e-2

Table 3.2: Estimation results, n=322

this works out to 18.1 cents a mile. However, this is without congestion. The congestion parameter is less intuitive, and needs to be considered along side the total congestion in the city. Figure 3.4 shows the marginal commuting cost as a function of the distance from the center of the city. Notice that congestion has a significant effect on marginal commuting cost. Both marginal cost and congestion peak about five miles from the center of the city. Using this cost, along with the distribution of commuters, the average commuting cost is \$111.2 per mile per year, and assuming 260 work days a year, this corresponds to 21.3 cents per mile. The interpretation is that congestion adds 17.7 percent to commuting costs, over a simple distance cost.

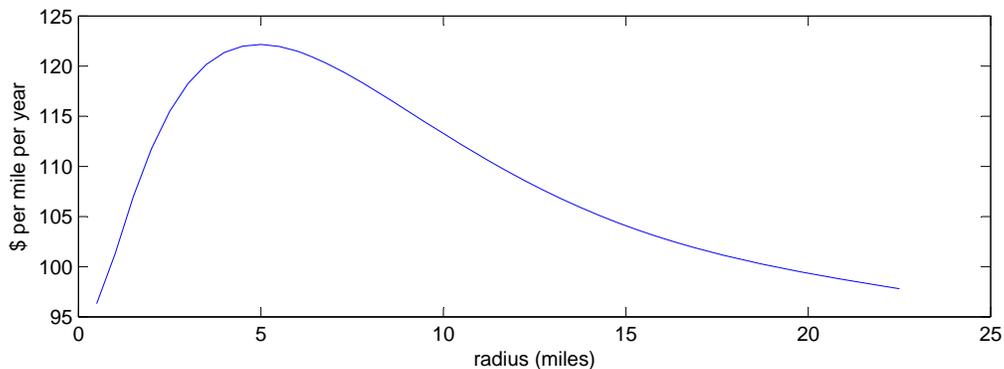


Figure 3.4: Marginal commuting cost as a function of radius from the city center

Finally, δ , the externality attenuation parameter, should be discussed. The interpretation of this parameter is as the coefficient in an exponential decay function with units in miles. Doing the math, this corresponds to a forty-four percent attenuation per mile.¹¹ The locational productivity gradient depends on the entire distribution of employment in the city, but this potentially explains why employment is found to very locally clustered within urban areas relative to residential density. Residential density gradients are dependent on transportation costs, which we would expect to have a more linear form.

Overall, the theoretical model produces a good fit of the underlying data. Table 3.3 shows population densities, employment densities, and commuting costs for the four locations used in the estimation. Again, these locations could be interpreted as the urban, suburban, and exurban regions of the metropolitan area. Both the population and employment densities are very consistent with the data, although, the model tends to place higher residential density in the city center than is present in the data.

This data is also presented in graphical form in Figure 3.5. Again the theory is consistent with the data. The one noticeable difference is that the theoretical population density gradients are more concave than the data, suggesting that the functional form of utility or the land services specification may need to be revisited. Also, the central business district displays considerably more employment and less residence than the model is able to explain, although it should be noted that the total area of the central business district is small compared to the total size of the region, so it does not skew the results as much as is depicted graphically.

3.7 Policy Considerations

This section considers the effects of transportation technologies on the spatial structure of the city. Before demonstrating these effects, it is important to note that these policy experiments are only relevant on a marginal level and for the initial allocations studied

¹¹This is consistent with Rosenthal and Strange (2003) who use a slightly different geography, but find that most of the advantages of production spillovers dissipate within a few miles.

	0 to 5 mi	5 to 9 mi	9 to 12 mi	12 to 23 mi
job density (per sq. mi.)	3,672 (3,518)	1,243 (1,185)	460 (768)	87 (86)
pop. density (per sq. mi.)	2,784 (1,989)	1,098 (1,263)	492 (759)	119 (130)
avg. commuting costs(\$)	1,462 (1,492)	1,563 (1,591)	1,669 (1,637)	1,875 (1,878)

Table 3.3: Fit of the model: Computational densities and average commuting costs by location, (equivalents from data in parenthesis)

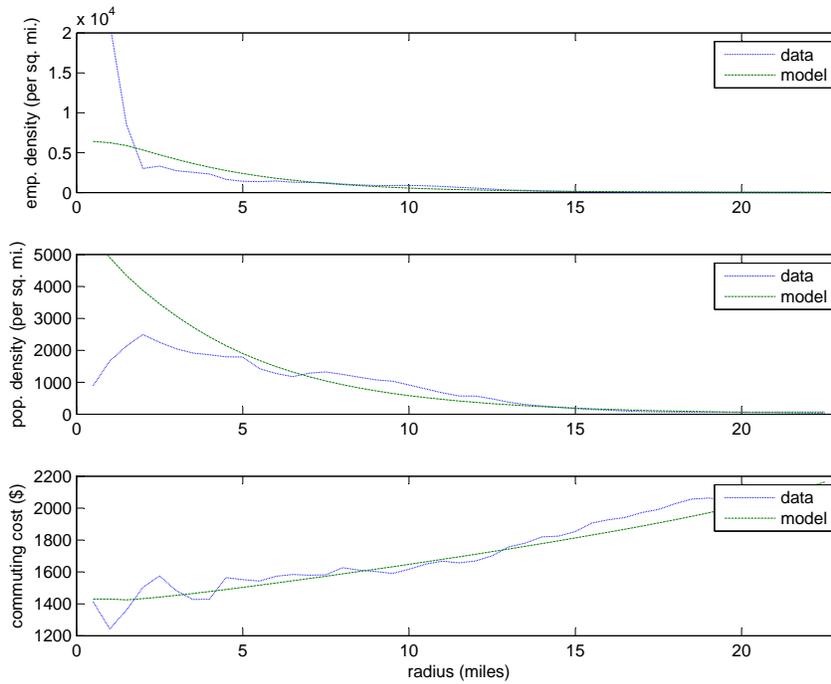


Figure 3.5: Density (per square mile) and commuting costs (dollars per year by residential location) as a function of radius from the city center: theoretical model and data

here. Agglomeration economies are, by nature, highly subject to initial conditions. A trivial example is in the case where there is no economic activity to begin with. In this case, the equilibrium solution is zero density everywhere, and this will not change with policy interventions of the sort discussed here. In other words, for agglomerations to emerge, there needs to be a seed of sorts. For this reason, these results should be considered in the context of a city which has similar characteristics to the one studied here, (i.e. a single business district with decreasing density moving away from the center.) Cities with multiple business districts or other geographic constraints may adjust differently in the face of transportation policy changes.

One way to examine the influence of transportation technologies on spatial structure is to hold all other parameters constant, adjust the transportation technologies, and analyze their effects. The goal of these experiments is not to analyze an increase or decrease in total transportation provision, but instead to understand how the mix of transportation provision changes the structure of the city. For example, we could conjecture that transit increases distance costs, but decreases congestion costs compared to automobile infrastructure. While it would be ideal to study the welfare ramifications of different transportation provision, a welfare comparison requires a complete understanding of the costs of providing different transportation networks. This would be a significant undertaking, and is outside the scope of this paper. Instead, the policy experiments will be analyzed in terms of the effect on relative spatial allocations of jobs and employment, as well as changes in the wage gradient.

In order to maintain consistency, all policies discussed are employment neutral, meaning that total employment in the city remains unchanged. In order to maintain employment levels, if we raise the distance cost, we must lower the congestion cost, corresponding to an increase in transit spending at the expense of highways. Furthermore, we will consider the initial estimates to be a baseline model from a purely automobile oriented city, given the low transit provision in Columbus. Policies demonstrated here then will incrementally increase distance costs and reduce congestion costs to illustrate the effect of gradually increasing transit provision.

Table 3.4 shows the effects of changing the transportation technology on the relative allocations of employment and residential population. The four locations can be interpreted as the urban, inner-suburban, suburban and exurban regions of the metropolitan area. In general, decreasing congestion cost leads to higher commercial density in the center of the city, and increased residential densities in the inner suburbs. In addition, we observe higher commercial density in the exurban region and lower residential density.

	baseline	policy 1	policy 2	policy 3
τ	94.9	100	105	120
κ	0.0117	0.0087	0.00576	0.00278
emp. density 0-5 mi.	3,672	3,681	3,685	3,686
emp. density 5-9 mi.	1,243	1,239	1,233	1,223
emp. density 9-12 mi.	460	456	451	444
emp. density 12-23 mi.	87	88	90	92
pop. density 0-5 mi.	2,784	2,780	2,774	2,764
pop. density 5-12 mi.	1,098	1,108	1,119	1,133
pop. density 9-12 mi.	492	499	507	515
pop. density 12-23 mi.	119	117	114	111

Table 3.4: Changes in employment and population densities for different transportation technologies (Total employment held constant)

To illustrate the changes in spatial distribution graphically, The top panel of Figure 3.6 shows the change in employment density plotted against radius from the city center, while the middle panel shows the change in residential density. This graph reveals that the employment density becomes relatively more concentrated at the center and edge of the city, while the residential density concentrates in the suburbs in between. This result has an intuitive interpretation. Because the distance cost has increased, workers move closer to the center to decrease the distance they must commute. In addition, workers are willing to travel through more congested areas, allowing jobs to be more highly concentrated. This concentration increases productivity for firms due to the agglomeration externality in concentrated areas, increasing wages paid relative to the baseline model in those locations. The change in commuting time located in the bottom panel further supports this interpretation, given that commuting costs rise significantly in the exurban region. It is interesting to note that the changes in allocations move closer to those observed in the more transit-oriented

Philadelphia (Figure 3.6 in section 2), which exhibits both a higher density central business district, and very dense inner ring suburbs.

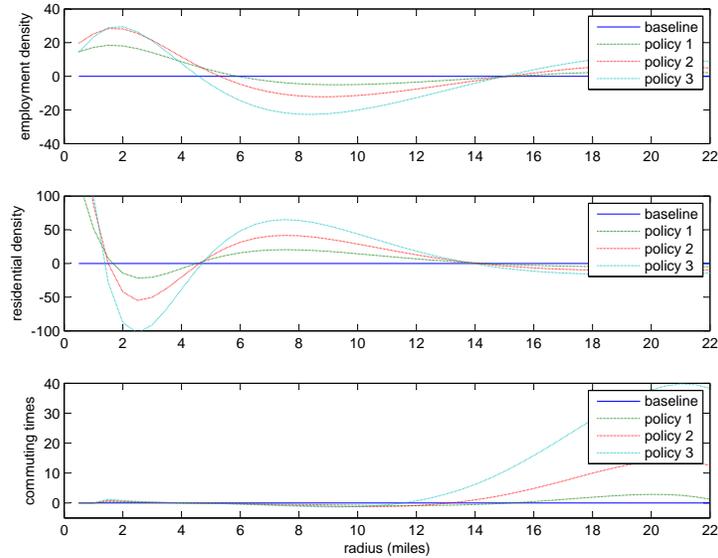


Figure 3.6: Change in employment density, residential density, and commuting costs relative to baseline for different transportation technologies

3.8 Conclusions

This paper has modeled and analyzed the effects of different transportation technologies on the spatial structure of cities in the presence of agglomeration externalities and commuting costs. A full general equilibrium model has been specified including a computational algorithm to solve for equilibrium for the parameterized model. Data was presented which illustrates the relationship of population and employment density to wages and commuting times. The data was used to estimate the model and found several important results. First, the model is able to explain the observed allocations of residential and employment location in the city, as well as commuting times.

Additionally, the effect of congestion is significant, and therefore, the provision of different types of transportation technologies can affect the overall structure of the city. Policy

experiments show that decreasing congestion costs relative to distance costs, holding total employment constant, increases the relative intensity of both the central business district and exurban areas. In addition, residential densities increase in inner ring suburbs as workers move to avoid commuting long distances. The interpretation of the results is that reducing congestion costs makes workers more willing to commute to high density areas, and therefore firms can take advantage of production externalities gained by locating in close proximity.

While these results give insight into how transportation costs affect the spatial allocation workers and jobs, I refrain from making any statements about the welfare ramifications. The provision of transportation networks is not free, and therefore total welfare is dependent on these costs. In fact, welfare is maximized when transportation costs are zero. Further study on the costs and provision of transportation must be incorporated to fully understand the optimal policy prescriptions for urban transportation. Nonetheless, the current research is a step toward a general understanding of how resources should be allocated to provide the best access and efficiency in a city, and provides a framework for enhanced analysis of a wide range of policy prescriptions.

Bibliography

- Adsera, A. (2000). Sectoral spillovers and the price of land: a cost analysis. *Regional Science and Urban Economics*, 30.
- Aguirregabiria, V., & Mira, P. (2007). Sequential estimation of dynamic discrete games. *Econometrica*, 75(1).
- Alonso, W. (1964). *Location and Land Use: Toward a General Theory of Land Rent*. Harvard University Press.
- Anas, A., & Kim, I. (1996). General equilibrium models of polycentric urban land use with endogenous congestion and job agglomeration. *Journal of Urban Economics*, 40(2).
- Bejari, P., Benkard, C. L., & Levin, J. (2007). Estimating dynamic models of imperfect competition. *Econometrica*, 75(5).
- Benkard, C. L. (2004). A dynamic analysis of the market for wide-bodied commercial aircraft. *Review of Economic Studies*, 71.
- Bresnahan, T. F., & Reiss, P. C. (1991). Entry and competition in concentrated markets. *The Journal of Political Economy*, 99(5).
- Brinkman, J., Coen-Pirani, D., & Sieg, H. (2010). Agglomeration externalities and the dynamics of firm location choices. *Working Paper*.
- Caselli, F., & Coleman, W. J. (2001). The u.s. structural transformation and regional convergence: A reinterpretation. *The Journal of Political Economy*, 109.

- Combes, P. P., Duranton, G., Gobillon, L., Puga, D., & Roux, S. (2010). The productivity advantages of large markets: Distinguishing agglomeration from firm selection. *Working Paper*.
- Daraszelski, U., & Pakes, A. (2006). A framework for applied dynamic analysis in io. *Handbook for Industrial Organization*, 3.
- Davis, J. C., & Henderson, J. V. (2008). The agglomeration of headquarters. *Regional Science and Urban Economics*, 38(5).
- Deckle, R., & Eaton, J. (1993). Agglomeration and land rents: Evidence from the prefectures. *The Journal of Urban Economics*, 46.
- Dixit, A. (1973). The optimum factory town. *The Bell Journal of Economics and Management Science*, 4(2).
- Dumais, G., Ellison, G., & Glaeser, E. (2002). Geographic concentration as a dynamic process. *Review of Economics and Statistics*, 84(2).
- Duranton, G., & Puga, D. (2001). Nursery cities: Urban diversity, process innovation, and the life cycle of products. *The American Economic Review*, 91(5).
- Duranton, G., & Puga, D. (2004). Microfoundations of urban agglomeration economies. *The Handbook of Regional and Urban Economics*, 4.
- Duranton, G., & Puga, D. (2005). From sectoral to functional urban specialisation. *Journal of Urban Economics*, 57(2).
- Ellison, G., & Glaeser, E. (1997). Geographic concentration in u.s. manufacturing industries: A dartboard approach. *Journal of Political Economy*, 105(5).
- Ericson, R., & Pakes, A. (1995). Markov-perfect industry dynamics: A framework fo empirical work. *Review of Economic Studies*, 62(1).
- Fujita, M., & Thisse, J.-F. (2002). *Economics of Agglomeration: Cities Industrial Location, and Regional Growth*. Cambridge.

- Gallant, A. R., & Nychka, D. W. (1987). Semi-nonparametric maximum likelihood estimation. *Econometrica*, 55(2).
- Gallant, A. R., & Tauchen, G. (1987). Semiparametric estimation of conditionally constrained heterogeneous processes: Asset pricing applications. *Econometrica*, 57(5).
- Gourieroux, C., & Monfort, A. (1992). Simulation based inference: A survey with special reference to panel data models. *Journal of Econometrics*, 59.
- Hansen, L. P. (1982). Large sample properties of generalized method of moments estimators. *Econometrica*, 50.
- Henderson, V., Kunkoro, A., & Turner, M. (1995). Industrial development in cities. *Journal of Political Economy*, 103(5).
- Holmes, T. J. (2011). The diffusion of wal-mart and economies of density. *Econometrica*, 79.
- Holmes, T. J., & Stevens, J. J. (2002). Geographic concentration and establishment scale. *Review of Economics and Statistics*, 84(4).
- Hopenhayn, H. (1992). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica*, 60(5).
- Hopenhayn, H., & Rogerson, R. (1993). Job turnover and policy evaluation: A general equilibrium analysis. *Journal of Political Economy*, 101.
- Hotelling, H. (1929). Stability in competition. *The Economic Journal*, 39(153).
- Jacobs, J. (1969). *The Economy of Cities*. Random House.
- Klette, Tor Jacob; Kortum, S. (2004). Innovating firms and aggregate innovation. *Journal of Political Economy*, 112(5).
- Krugman, P. (1991). Increasing returns and economic geography. *Journal of Political Economy*, 99(3).

- Lucas, R. E., & Rossi-Hansberg, E. (2002). On the internal structure of cities. *Econometrica*, 70(4).
- Marshall, A. (1890). *Principles of Economics*. Macmillian.
- Melitz, M. J. (2003). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6).
- Mills, E. S. (1967). Transportation and patterns of urban development: An aggregative model of resource allocation in a metropolitan area. *The American Economic Review*, 57(2).
- Muth, R. F. (1969). *Cities and Housing: The Spatial Pattern of Urban Residential Land Use*. University of Chicago Press.
- Pakes, A., & McGuire, P. (1994). Computing markov perfect nash equilibria: Numerical implications of a dynamic differentiated product model. *Rand Journal of Economics*, 25(5).
- Roback, J. (1982). Wages, rents, and the quality of life. *Journal of Political Economy*, 90.
- Rosenthal, S. S., & Strange, W. C. (2001). The determinants of agglomeration. *The Journal of Urban Economics*, 50(2).
- Rosenthal, S. S., & Strange, W. C. (2003). Geography, industrial organization, and agglomeration. *Review of Economics and Statistics*, 85(2).
- Rosenthal, S. S., & Strange, W. C. (2004). Evidence on the nature and sources of agglomeration economies. *The Handbook of Regional and Urban Economics*, 4.
- Rossi-Hansberg, E. (2004). Optimal urban land use and zoning. *Review of Economic Dynamics*, 7.
- Rossi-Hansberg, E., & Wright, M. L. J. (2007). Establishment size dynamics in the aggregate economy. *American Economic Review*, 97(5).

- Small, K. A. (1983). The incidence of congestion tolls on urban highways. *Journal of Urban Economics*, 13.
- Solow, R. M. (1972). Congestion, density and the use of land in transportation. *Swedish Journal of Economics*, 74.
- Tse, C. Y., & Chan, A. W. (2003). Estimating the commuting cost and commuting time propoerty price gradients. *Regional Science and Urban Economics*, 33.
- Von Thünen, J. H. (1826). *The Isolated State*. Oxford.
- Wheaton, W. (1999). Real estate cycles: Some fundamentals. *Real Estate Economics*, 27.
- Wheaton, W. C. (2004). Commuting, congestion, and employment dispersal, in cities with mixed land use. *The Journal of Urban Economics*, 55.